

Package ‘Delaporte’

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Type Package

Title Statistical Functions for the Delaporte Distribution

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Description Provides probability mass, distribution, quantile, random-variate generation, and method-of-moments parameter-estimation functions for the Delaporte distribution with parameterization based on Vose (2008) <isbn:9780470512845>. The Delaporte is a discrete probability distribution which can be considered the convolution of a negative binomial distribution with a Poisson distribution. Alternatively, it can be considered a counting distribution with both Poisson and negative binomial components. It has been studied in actuarial science as a frequency distribution which has more variability than the Poisson, but less than the negative binomial.

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URL <https://github.com/aadler/Delaporte>

BugReports <https://github.com/aadler/Delaporte/issues>

Depends R (>= 3.6.0)

Imports stats

Suggests covr, tinytest

ByteCompile yes

NeedsCompilation yes

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Delaporte-package *Statistical Functions for the Delaporte Distribution*

Description

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Details

The DESCRIPTION file:

```
Package:           Delaporte
Type:             Package
Title:           Statistical Functions for the Delaporte Distribution
Version:         8.0.3
Date:            2022-09-05
Authors@R:       c(person(given="Avraham", family="Adler", role=c("aut", "cph", "cre"), email="Avraham.
Description:     Provides probability mass, distribution, quantile, random-variate generation, and method-of
License:         BSD_2_clause + file LICENSE
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Suggests:        covr, tinytest
ByteCompile:     yes
NeedsCompilation: yes
Author:          Avraham Adler [aut, cph, cre] (<https://orcid.org/0000-0002-3039-0703>)
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```

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Delaporte-defunct   Defunct functions in package 'Delaporte'
Delaporte-package   Statistical Functions for the Delaporte
                    Distribution
```

Author(s)

NA

Maintainer: NA

Delaporte

*The Delaporte Distribution***Description**

Density, distribution, quantile, random variate generation, and method of moments parameter estimation functions for the Delaporte distribution with parameters alpha, beta, and lambda.

Usage

```
ddelap(x, alpha, beta, lambda, log = FALSE)
pdelap(q, alpha, beta, lambda, lower.tail = TRUE, log.p = FALSE)
qdelap(p, alpha, beta, lambda, lower.tail = TRUE, log.p = FALSE, exact=TRUE)
rdelap(n, alpha, beta, lambda, exact = TRUE)
```

```
MoMdelap(x, type = 2L)
```

Arguments

x	vector of (non-negative integer) quantiles.
q	vector of quantiles.
p	vector of probabilities.
n	number of observations.
alpha	vector of alpha parameters of the gamma portion of the Delaporte distribution. Must be strictly positive, but need not be integer.
beta	vector of beta parameters of the gamma portion of the Delaporte distribution. Must be strictly positive, but need not be integer.
lambda	vector of lambda parameters of the Poisson portion of the Delaporte distribution. Must be strictly positive, but need not be integer.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$, otherwise, $P[X > x]$.
exact	logical; if TRUE uses double summation to generate quantiles or random variates. Otherwise uses Poisson-negative binomial approximation.
type	integer; 1L will return g1 , 2L will return G1 , and 3L will return b1 , as per skewness .

Details

The Delaporte distribution with parameters alpha, beta, and lambda is a discrete probability distribution which can be considered the convolution of a negative binomial distribution with a Poisson distribution. Alternatively, it can be considered a counting distribution with both Poisson and negative binomial components. The Delaporte's probability mass function, called via `dde lap`, is:

$$p(n) = \sum_{i=0}^n \frac{\Gamma(\alpha + i)\beta^i \lambda^{n-i} e^{-\lambda}}{\Gamma(\alpha) i! (1 + \beta)^{\alpha+i} (n - i)!}$$

for $n = 0, 1, 2, \dots; \alpha, \beta, \lambda > 0$.

If an element of `x` is not integer, the result of `dde lap` is zero with a warning.

The Delaporte's cumulative distribution function, `pde lap`, is calculated through double summation:

$$CDF(n) = \sum_{j=0}^n \sum_{i=0}^j \frac{\Gamma(\alpha + i)\beta^i \lambda^{j-i} e^{-\lambda}}{\Gamma(\alpha) i! (1 + \beta)^{\alpha+i} (j - i)!}$$

for $n = 0, 1, 2, \dots; \alpha, \beta, \lambda > 0$. If only singleton values for the parameters are passed in, the function uses a shortcut. It identifies the largest value passed to it, computes a vector of CDF values for all integers up to and including that value, and reads the remaining results from this vector. This requires only one double summation instead of `length(q)` such summations. If at least one of the parameters is itself a vector of length greater than 1, the function has to build the double summation for each entry in `q`.

`dde lap` will return 0 for all values $> 2^{31}$ whereas `pde lap` will not run at all, due to the limitations of integer representation. Also, for values $> 2^{15}$, `pde lap` will ask for positive input from the user to continue, as otherwise, depending on the parameters, the function can take hours to complete given its double-summation nature.

The quantile function, `qde lap`, is right continuous: `qde lap(q, alpha, beta, lambda)` is the smallest integer x such that $P(X \leq x) \geq q$. This function has two versions. When `exact = TRUE`, the function builds a CDF vector and the first value for which the CDF is greater than or equal to `q` is returned as the quantile. While this procedure is accurate, for sufficiently large α, β , or λ it can take a very long time. Therefore, when dealing with singleton parameters, `exact = FALSE` can be passed to take advantage of the Delaporte's definition as a counting distribution with both a Poisson and a negative binomial component. Based on Karlis & Xekalaki (2005) it will generate n gamma variates Γ with shape α and scale β and then n pseudo-Delaporte variates as Poisson random variables with parameter $\lambda + \Gamma$, finally calling the `quantile` function on the result. The "exact" method is always more accurate and is also significantly faster for reasonable values of the parameters. Also, the "exact" method must be used when passing parameter vectors, as the pooling would become intractable. Ad-hoc testing indicates that the "exact" method should always be used until $\alpha\beta + \lambda \approx 2500$. Both versions return `NaN` for quantiles < 0 , 0 for quantiles $= 0$, and `Inf` for quantiles ≥ 1 .

The random variate generator, `rde lap`, also has multiple versions. When `exact = TRUE`, it uses inversion by creating a vector of n uniformly distributed random variates between 0 and 1. If all the parameters are singletons, a single CDF vector is constructed as per the quantile function, and the entries corresponding to the uniform variates are read off of the constructed vector. If the parameters are themselves vectors, then it passes the entire uniform variate vector to `qde lap`, which is slower.

When `exact = FALSE`, regardless of the length of the parameters, it generates n gamma variates Γ with shape α and scale β and then n pseudo-Delaporte variates as Poisson random variables with parameter $\lambda + \Gamma$. As there is no pooling, each individual random variate reflects the parameter triplet which generated it. The non-inversion method is usually faster.

`MoMdelap` uses the definition of the Delaporte's mean, variance, and skew to calculate the method of moments estimates of α , β , and λ , which it returns as a numeric vector. This estimate is also a reasonable starting point for maximum likelihood estimation using nonlinear optimizers such as `optim` or `nloptr`. If the data is clustered near 0, there are times when method of moments would result in a non-positive parameter. In these cases `MoMdelap` will throw an error. For the sample skew, the user has the choice to select `g1`, `G1`, or `b1` as defined in Joanes & Gill (1997) and found in `skewness`. The selection defaults to option 2, `G1`, which Joanes & Gill found to have the least mean-square error for non-normal distributions.

Value

`dodelap` gives the probability mass function, `podelap` gives the cumulative distribution function, `qodelap` gives the quantile function, and `rodelap` generates random deviates. Values close to 0 (less than or equal to machine epsilon) for α , β or λ will return NaN for that particular entry. Proper triplets within a set of vectors should still return valid values. For the approximate versions of `qodelap` and `rodelap`, having a value close to 0 will trip an error, sending the user to the exact version which currently properly handles vector-based inputs which contain 0.

Invalid quantiles passed to `qodelap` will result in return values of NaN or Inf as appropriate.

The length of the result is determined by `x` for `dodelap`, `q` for `podelap`, `p` for `qodelap`, and `n` for `rodelap`. The distributional parameters (α , β , λ) are recycled as necessary to the length of the result.

When using the lower `.tail = FALSE` or `log / log.p = TRUE` options, some accuracy may be lost at knot points or the tail ends of the distributions due to the limitations of floating point representation.

`MoMdelap` returns a triplet comprising a method-of-moments based estimate of α , β , and λ .

Author(s)

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References

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- Karlis, D. and Xekalaki, E. (2005) Mixed Poisson Distributions. *International Statistical Review* **73**(1), 35–58. <https://projecteuclid.org/euclid.isr/1112304811>
- Vose, D. (2008) *Risk analysis: a quantitative guide* (Third, illustrated ed.). John Wiley & Sons. pp. 618–619 ISBN 978-0-470-51284-5

See Also

`Distributions` for standard distributions, including `dnbinom` for the negative binomial distribution and `dpois` for the Poisson distribution, and `skewness` for skew options.

Examples

```

## Density and distribution
A <- c(0, seq_len(50))
PMF <- ddelap(A, alpha = 3, beta = 4, lambda = 10)
CDF <- pdelap(A, alpha = 3, beta = 4, lambda = 10)

##Quantile
A <- seq(0,.95, .05)
qdelap(A, alpha = 3, beta = 4, lambda = 10)
A <- c(-1, A, 1, 2)
qdelap(A, alpha = 3, beta = 4, lambda = 10)

## Compare a Poisson, negative binomial, and three Delaporte distributions with the same mean:
P <- rpois(25000, 25) ## Will have the tightest spread
DP1 <- rdelap(10000, alpha = 2, beta = 2, lambda = 21) ## Close to the Poisson
DP2 <- rdelap(10000, alpha = 3, beta = 4, lambda = 13) ## In between
DP3 <- rdelap(10000, alpha = 4, beta = 5, lambda = 5) ## Close to the Negative Binomial
NB <- rnbinom(10000, size = 5, mu = 25) ## Will have the widest spread
mean(P);mean(NB);mean(DP1);mean(DP2);mean(DP3) ## Means should all be near 25
MoMdelap(DP1);MoMdelap(DP2);MoMdelap(DP3) ## Estimates should be close to originals
## Not run:
plot(density(P), col = "black", lwd = 2, main = "Distribution Comparison",
     xlab = "Value", xlim = c(0, 80))
lines(density(DP1), col = "blue", lwd = 2)
lines(density(DP2), col = "green3", lwd = 2)
lines(density(DP3), col = "orange3", lwd = 2)
lines(density(NB), col = "red", lwd = 2)
legend(x = "topright", legend = c("Poisson {l=25}", "DP {a=2, b=2, l=21}",
 "DP {a=3, b=4, l=13}", "DP {a=4, b=5, l=5}", "NegBinom {a=5, b=5}"),
      col=c("black", "blue", "green3", "orange3", "red"), lwd = 2)

## End(Not run)

```

Delaporte-defunct

Defunct functions in package 'Delaporte'

Description

These functions are defunct and no longer available.

Details

The following functions are defunct; use the replacement indicated below:

- As of version 5.0.0, the `old = TRUE` method for the `qdelap` and `rdelap` functions is defunct and has been removed.

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