# Package 'RcppAlgos' 

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Title High Performance Tools for Combinatorics and Computational Mathematics

Description Provides optimized functions and flexible combinatorial iterators implemented in $\mathrm{C}++$ for solving problems in combinatorics and computational mathematics. Utilizes the RMatrix class from 'RcppParallel' for thread safety. There are combination/permutation functions with constraint parameters that allow for generation of all results of a vector meeting specific criteria (e.g. generating integer partitions/compositions or finding all combinations such that the sum is between two bounds). Capable of generating specific combinations/permutations (e.g. retrieve only the nth lexicographical result) which sets up nicely for parallelization as well as random sampling. Gmp support permits exploration where the total number of results is large (e.g. comboSample(10000, 500, $\mathrm{n}=4)$ ). Additionally, there are several high performance number theoretic functions that are useful for problems common in computational mathematics. Some of these functions make use of the fast integer division library 'libdivide'. The primeSieve function is based on the segmented sieve of Eratosthenes implementation by Kim Walisch. It is also efficient for large numbers by using the cache friendly improvements originally developed by Tomás Oliveira. Finally, there is a prime counting function that implements Legendre's formula based on the work of Kim Walisch.

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http://sweet.ua.pt/tos/software/prime_sieve.html
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RcppAlgos-package High Performance Tools for Combinatorics and Computational Mathematics

## Description

The RcppAlgos package attacks age-old problems in combinatorics and computational mathematics.

## Goals

1. The main goal is to encourage fresh and creative approaches to foundational problems. The question that most appropriately summarizes RcppAlgos is: "Can we do better?".
2. Provide highly optimized functions that facilitates a broader spectrum of researchable cases. E. $g$

- Investigating the structure of large numbers over wide ranges:
- primeFactorizeSieve (10^13-10^7, 10^13 + 10^7)
- primeSieve(2^53-10^10, 2^53-1, nThreads = 32)
- Easily explore combinations/permutations/partitions that would otherwise be inaccessible due to time of execution/memory constraints:

```
- c_iter = comboIter(10000, 100)
    bigSamp = gmp::urand.bigz(3, gmp::log2.bigz(comboCount(10000, 100)))
    iter[[bigSamp]]
- p_iter = partitionsIter(5000, 100, target = 6000)
    p_iter[[1e9]] ## start iterating from index = 1e9
    p_iter@nextIter()
    p_iter@nextNIter(1e3)
- comboGeneral(150, 5, constraintFun = "sum", Parallel = TRUE)
- parallel::mclapply(seq(...), function(x) {
    temp = permuteGeneral(15, 10, lower = x, upper = y)
    ## analyze permutations
    ## output results
        }, mc.cores = detectCores() - 1))
- partitionsGeneral(0:80, repetition = TRUE)
- permuteSample(rnorm(100), 10, freqs = rep(1:4, 25), n=15, seed=123)
- set.seed(123)
    comboGeneral(runif(42, 0, 50), 10,
                                    constraintFun = "mean",
                                    comparisonFun = c(">","<"),
                limitConstraints = c(39.876, 42.123))
```

3. Speed!!!!... You will find that the functions in RcppAlgos are some of the fastest of their type available in R .

## Author(s)

Joseph Wood

## Description

The Combo class family are S4-classes that expose C++ classes that provide access to iterators and other useful methods.

## Slots

of "Combo" and all classes inheriting from it:
nextIter Retrieve the next lexicographical result
nextNIter Pass an integer $n$ to retrieve the next $n$ lexicographical results
nextRemaining Retrieve all remaining lexicographical results
currIter Returns the current iteration
prevIter Retrieve the previous lexicographical result (the next reverse lexicographical result)
prevNIter Pass an integer $n$ to retrieve the previous $n$ lexicographical results (the next $n$ reverse lexicographical results)
prevRemaining Retrieve all remaining reverse lexicographical results
startOver Resets the iterator
sourceVector View the source vector
summary Returns a list of summary information about the iterator
front Retrieve the first lexicographical result
back Retrieve the last lexicographical result
randomAccess Random access method. Pass a single value or a vector of valid indices. If a single value is passed, the internal index of the iterator will be updated, however if a vector is passed the internal state will not change. GMP support allows for flexible indexing.

## Author(s)

Joseph Wood

## See Also

Partitions-class, Constraints-class

## Examples

showClass("Combo")

## Description

Calculate the number of combinations/permutations of a vector chosen $m$ at a time with or without replacement. Additionally, these functions can calculate the number of combinations/permutations of multisets.

## Usage

comboCount (v, m = NULL, repetition = FALSE, freqs = NULL)
permuteCount(v, m = NULL, repetition = FALSE, freqs = NULL)

## Arguments

$v \quad$ Source vector. If $v$ is a positive integer, it will be converted to the sequence $1: v$. If $v$ is a negative integer, it will be converted to the sequence $v:-1$. All atomic types are supported (See is.atomic).
m
Number of elements to choose. If repetition = TRUE or freqs is utilized, $m$ can exceed the length of $v$. If $m=N U L L$, the length will default to length( $v$ ) or sum(freqs).
repetition Logical value indicating whether combinations/permutations should be with or without repetition. The default is FALSE.
freqs A vector of frequencies used for producing all combinations/permutations of a multiset of $v$. Each element of freqs represents how many times each element of the source vector, $v$, is repeated. It is analogous to the times argument in rep. The default value is NULL.

## Value

A numerical value representing the total number of combinations/permutations.

## Note

When the number of results exceeds $2^{53}-1$, a number of class bigz is returned.

## See Also

comboGeneral, permuteGeneral

## Examples

```
## Same interface as the respective "general" functions:
## i.e. comboGeneral & permuteGeneral
permuteCount(-5)
permuteCount(5)
comboCount(25, 12)
permuteCount(15, 7, TRUE)
comboCount(25, 12, freqs = rep(2, 25))
## Return object of class 'bigz'
comboCount(250, 15, freqs = rep(2, 250))
```

comboGeneral Generate Combinations and Permutations of a Vector with/without Constraints

## Description

- Generate combinations or permutations of a vector with or without constraints.
- Produce results in parallel using the Parallel or nThreads arguments. You can also apply each of the five compiled functions given by the argument constraintFun in parallel.
- The arguments lower and upper make it possible to generate combinations/permutations in chunks allowing for parallelization via the parallel-package. This is convenient when you want to apply a custom function to the output in parallel.
- Attack integer partition and general subset sum problems.
- GMP support allows for exploration of combinations/permutations of vectors with many elements.
- The output is in lexicographical order.


## Usage

## Arguments

V
m
repetition
freqs
lower The lower bound. Combinations/permutations are generated lexicographically, thus utilizing this argument will determine which specific combination/permutation to start generating from (e.g. comboGeneral $(5,3$, lower $=6)$ is equivalent to comboGeneral $(5,3)[6: \operatorname{choose}(5,3)]$,$) . This argument along with upper$ is very useful for generating combinations/permutations in chunks allowing for easy parallelization.
upper The upper bound. Similar to lower, however this parameter allows the user to stop generation at a specific combination/permutation (e.g. comboGeneral(5, 3 , upper $=5$ ) is equivalent to comboGeneral $(5,3)[1: 5]$,
If the output is constrained and lower isn't supplied, upper serves as a cap for how many results will be returned that meet the criteria (e.g. setting upper $=$ 100 alone will return the first 100 results that meet the criteria, while setting lower $=1$ and upper $=100$ will test the first 100 results against the criteria).
In addition to the benefits listed for lower, this parameter is useful when the total number of combinations/permutations without constraint is large and you expect/need a small number of combinations/permutations that meet a certain criteria. Using upper can improve run time if used judiciously as we call the member function reserve of std::vector. See examples below.
constraintFun Function to be applied to the elements of $v$ that should be passed as a string (e.g. constraintFun = "sum"). The possible constraint functions are: "sum", "prod", "mean", "max", \& "min". The default is NULL, meaning no function is applied.
comparisonFun Comparison operator that will be used to compare limitConstraints with the result of constraintFun applied to $v$. It should be passed as a string or a vector of two strings (e.g. comparisonFun = "<=" or comparisonFun = c (">", "<")). The possible comparison operators are: "<", ">", "<=", ">=", "==". The default is NULL.
When comparisonFun is a vector of two comparison strings, e.g comparisonFun $=\mathrm{c}$ (comp1, comp2), and limitConstraints is a vector of two numerical values, e.g limitConstraints $=c(x 1, x 2)$, the combinations/permutations will be filtered in one of the following two ways:

1. When comp1 is one of the 'greater-than' operators (i.e. " $>=$ " or " $>$ "), comp2 is one of the 'less-than' operators (i.e. " $<==$ " or "<"), and x1 <x2, the combinations/permutations that are returned will have a value (after constraintFun has been applied) between $\times 1$ and $\times 2$.
2. When comp1 and comp2 are defined as in \#1 and $x 1>x 2$, the combinations/permutations that are returned will have a value outside the range of $x 1$ and $\times 2$. See the examples below.
In other words, the first comparison operator is applied to the first limit and the second operator is applied to the second limit.
limitConstraints
This is the value(s) that will be used for comparison. Can be passed as a single value or a vector of two numerical values. The default is NULL. See the definition of comparisonFun as well as the examples below for more information.
keepResults A logical flag indicating if the result of constraintFun applied to $v$ should be displayed; if TRUE, an additional column of results will be added to the resulting matrix. The default is FALSE. If user is only applying constraintFun, keepResults will default to TRUE.
E.g. The following are equivalent and will produce a $4^{\text {th }}$ column of row sums:

- comboGeneral(5, 3 constraintFun = "sum", keepResults = TRUE)
- comboGeneral(5, 3 constraintFun = "sum")

FUN Function to be applied to each combination/permutation. The default is NULL.
Parallel Logical value indicating whether combinations/permutations should be generated in parallel using $n-1$ threads, where $n$ is the maximum number of threads. The default is FALSE. If nThreads is not NULL, it will be given preference (e.g. if user has 8 threads with Parallel $=$ TRUE and $n$ Threads $=4$, only 4 threads will be spawned). If your system is single-threaded, the arguments Parallel and $n$ Threads are ignored.
nThreads Specific number of threads to be used. The default is NULL. See Parallel.
tolerance A numeric value greater than or equal to zero. This parameter is utilized when a constraint is applied on a numeric vector. The default value is 0 when it can be determined that whole values are being utilized, otherwise it is sqrt (.Machine\$double.eps) which is approximately $1.5 e-8$. N.B. If the input vector is of type integer, this parameter will be ignored and strict equality will be enforced.
FUN.VALUE A template for the return value from FUN. See 'Details' of vapply for more information.

## Details

Finding all combinations/permutations with constraints is optimized by organizing them in such a way that when constraintFun is applied, a partially monotonic sequence is produced. Combinations/permutations are added successively, until a particular combination exceeds the given constraint value for a given constraint/comparison function combo. After this point, we can safely skip several combinations knowing that they will exceed the given constraint value.

When there are any negative values in v and constraintFun = "prod", producing a monotonic set is non-trivial for the general case. As a result, performance will suffer as all combinations/permutations must be tested against the constraint criteria.

## Value

- In general, a matrix with $m$ or $m+1$ columns, depending on the value of keepResults
- If FUN is utilized and FUN. VALUE = NULL, a list is returned
- When both FUN and FUN. VALUE are not NULL, the return is modeled after the return of vapply. See the 'Value' section of vapply.


## Note

- Parallel and nThreads will be ignored in the following cases:
- When the output is constrained (except for most partitions cases)
- If the class of the vector passed is character, raw, and complex (N.B. Rcpp: :CharacterMatrix is not thread safe). Alternatively, you can generate an indexing matrix in parallel.
- If FUN is utilized.
- If either constraintFun, comparisonFun or limitConstraints is NULL -or- if the class of the vector passed is logical, character, raw, factor, or complex, the constraint check will not be carried out. This is equivalent to simply finding all combinations/permutations of $v$ choose $m$.
- The maximum number of combinations/permutations that can be generated at one time is $2^{31}-$ 1. Utilizing lower and upper makes it possible to generate additional combinations/permutations.
- Factor vectors are accepted. Class and level attributes are preserved except when FUN is used.
- Lexicographical ordering isn't guaranteed for permutations if lower isn't supplied and the output is constrained.
- If lower is supplied and the output is constrained, the combinations/permutations that will be tested will be in the lexicographical range lower to upper or up to the total possible number of results if upper is not given. See the second paragraph for the definition of upper.
- FUN will be ignored if the constraint check is satisfied.


## Author(s)

Joseph Wood

## References

- Passing user-supplied C++ functions
- Monotonic Sequence
- Multiset
- Lexicographical Order
- Subset Sum Problem
- Partition (Number Theory)


## Examples

```
system.time(comboGeneral(17, 8))
system.time(permuteGeneral(13, 6))
system.time(comboGeneral(13,10,repetition = TRUE))
system.time(permuteGeneral(factor(letters[1:9]),6,TRUE))
## permutations of the multiset (with or w/o setting m) :
## c(1,1,1,1,2, 2, 2, 3, 3,4)
system.time(permuteGeneral(4, freqs = c(4,3,2,1)))
#### Examples using "upper" and "lower":
## Generate some random data
set.seed(1009)
mySamp = sort(rnorm(75, 997, 23))
permuteCount(75, 10, freqs = rep(1:3, 25))
# >Big Integer ('bigz') :
# >[1] 4606842576291495952
## See specific range of permutations
permuteGeneral(75, 10, freqs = rep(1:3, 25),
                lower = 1e12, upper = 1e12 + 10)
## Researcher only needs 1000 7-tuples of mySamp
## such that the sum is greater than 7200.
system.time(comboGeneral(mySamp, 7, FALSE, constraintFun = "sum",
    comparisonFun = ">", limitConstraints = 7200, upper = 1000))
## Similarly, you can use "lower" to obtain the last rows.
## Generate the last 10 rows
system.time(comboGeneral(mySamp, 7, lower = choose(75, 7) - 9))
## Or if you would like to generate a specific chunk,
## use both "lower" and "upper". E.g. Generate one
## million combinations starting with the 900,000,001
## lexicographic combination.
t1 = comboGeneral(mySamp, 7,
    lower = 9*10^8 + 1,
    upper = 9*10^8 + 10^6)
## class of the source vector is preserved
class(comboGeneral(5,3)[1,]) == class(1:5)
class(comboGeneral(c(1,2:5),3)[1,]) == class(c(1,2:5))
class(comboGeneral(factor(month.name),3)[1,]) == class(factor(month.name))
## Using keepResults will add a column of results
t2 = permuteGeneral(-3,6,TRUE,constraintFun = "prod",
                        keepResults = TRUE)
t3 = comboGeneral(-3,6,TRUE,constraintFun = "sum",
    comparisonFun = "==",
```

```
limitConstraints = -8,
keepResults = TRUE)
## Using multiple constraints:
## Get combinations such that the product
## is between 3000 and 4000 inclusive
comboGeneral(5, 7, TRUE, constraintFun = "prod",
    comparisonFun = c(">=","<="),
    limitConstraints = c(3000, 4000),
    keepResults = TRUE)
## Or, get the combinations such that the
## product is less than or equal to 10 or
## greater than or equal to 40000
comboGeneral(5, 7, TRUE, constraintFun = "prod",
    comparisonFun = c("<=",">="),
    limitConstraints = c(10, 40000),
    keepResults = TRUE)
#### General subset sum problem
set.seed(516781810)
comboGeneral(runif(100, 0, 42), 5, constraintFun = "mean",
    comparisonFun = "==", limitConstraints = 30,
    tolerance = 0.0000002)
#### Integer Partitions
comboGeneral(0:5, 5, TRUE, constraintFun = "sum",
    comparisonFun = "==", limitConstraints = 5)
## Using FUN
comboGeneral(10000, 5, lower = 20, upper = 22,
    FUN = function(x) {
            which(cummax(x) %% 2 == 1)
        })
## Not run:
## Parallel example generating more than 2^31-1 combinations.
library(parallel)
numCores = detectCores() - 1
## 10086780 evenly divides choose(35, 15) and is "small enough" to
## generate quickly in chunks.
system.time(mclapply(seq(1, comboCount(35, 15), 10086780), function(x) {
    a = comboGeneral(35, 15, lower = x, upper = x + 10086779)
    ## do something
    x
}, mc.cores = numCores))
## Find 13-tuple combinations of 1:25 such
```

```
## that the mean is less than 10
system.time(myComb <- comboGeneral(25, 13, FALSE,
    constraintFun = "mean",
    comparisonFun = "<",
        limitConstraints = 10))
## Alternatively, you must generate all combinations and subsequently
## subset to obtain the combinations that meet the criteria
system.time(myComb2 <- combn(25, 13))
system.time(myCols <- which(colMeans(myComb2) < 10))
system.time(myComb2 <- myComb2[, myCols])
## Any variation is much slower
system.time(myComb2 <- combn(25, 13)[,combn(25, 13, mean) < 10])
## Test equality with myComb above
all.equal(myComb, t(myComb2))
## much faster using Parallel = TRUE
system.time(permuteGeneral(15, 8, lower = 250000000, Parallel = TRUE))
system.time(permuteGeneral(15, 8, lower = 250000000))
system.time(comboGeneral(30, 10, Parallel = TRUE))
system.time(comboGeneral(30, 10))
## Parallel also works when applying constraintFun solely
system.time(comboGeneral(30, 10, Parallel = TRUE, constraintFun = "sum"))
system.time(comboGeneral(30, 10, constraintFun = "sum"))
## Depending on # of cores available, using Parallel with
## constraintFun is faster than rowSums or rowMeans alone
combs = comboGeneral(30, 10)
system.time(rowSums(combs))
## Fun example... see stackoverflow:
## https://stackoverflow.com/q/22218640/4408538
system.time(permuteGeneral(seq(0L,100L,10L),8,TRUE,
    constraintFun = "sum",
    comparisonFun = "==",
    limitConstraints = 100))
## End(Not run)
```

comboGrid

## Description

This function efficiently generates Cartesian-product-like output where order does not matter. It is loosely equivalent to the following:

- $t=$ expand.grid(list)
- $\mathrm{t}=\mathrm{t}[\mathrm{do} . \mathrm{call}($ order, t$), \mathrm{]}$
- key $=\operatorname{apply}(\mathrm{t}, 1$, function( x$)$ paste0 (sort $(\mathrm{x})$, collapse $=$ "") )
- t[!duplicated(key), ]


## Usage

comboGrid(..., repetition = TRUE)

## Arguments

| $\ldots$. | vectors, factors or a list containing these. (See ?expand.grid). |
| :--- | :--- |
| repetition | Logical value indicating whether results should be with or without repetition. <br>  <br>  <br> The default is TRUE. |

## Value

If items with different classes are passed, a data frame will be returned, otherwise a matrix will be returned.

## Author(s)

Joseph Wood

## Examples

```
## return a matrix
expGridNoOrder = comboGrid(1:5, 3:9, letters[1:5], letters[c(1,4,5,8)])
head(expGridNoOrder)
tail(expGridNoOrder)
expGridNoOrderNoRep = comboGrid(1:5, 3:9, letters[1:5],
                                    letters[c(1,4,5,8)], repetition = FALSE)
head(expGridNoOrderNoRep)
tail(expGridNoOrderNoRep)
```

comboGroups
Partition a Vector into Groups of Equal Size

## Description

- Generate partitions of a vector into groups of equal size. See Create Combinations in R by Groups on http://stackoverflow.com for a direct use case.
- Produce results in parallel using the Parallel or nThreads arguments.
- GMP support allows for exploration where the number of results is large.
- The output is in lexicographical order by groups.


## Usage

```
comboGroups(v, numGroups, retType = "matrix",
    lower = NULL, upper = NULL, Parallel = FALSE,
    nThreads = NULL)
```


## Arguments

v
numGroups
retType A string, "3Darray" or "matrix", that determines the shape of the output. The default is "matrix".
lower The lower bound. Partitions of groups are generated lexicographically, thus utilizing this argument will determine which specific result to start generating from (e.g. comboGroups $(8,2$, lower $=30)$ is equivalent to comboGroups ( 8 , 2) $[30$ : comboGroupsCount $(8,2)]$,$) . This argument along with upper is very$ useful for generating results in chunks allowing for easy parallelization.
upper The upper bound. Similar to lower, however this parameter allows the user to stop generation at a specific result (e.g. comboGroups ( 8,2 , upper $=5$ ) is equivalent to comboGroups $(8,2)[1: 5]$,
Parallel Logical value indicating whether results should be generated in parallel using $n-1$ threads, where $n$ is the maximum number of threads. The default is FALSE. If $n$ Threads is not NULL, it will be given preference (e.g. if user has 8 threads with Parallel $=$ TRUE and $n$ Threads $=4$, only 4 threads will be spawned). If your system is single-threaded, the arguments Parallel and nThreads are ignored.
nThreads Specific number of threads to be used. The default is NULL. See Parallel.

## Details

Conceptually, this problem can be viewed as generating all permutations of the vector $v$ and removing the within group permutations. To illustrate this, let us consider the case of generating partitions of $1: 8$ into 2 groups of size 4 .

- To begin, generate the permutations of $1: 8$ and group the first/last four elements of each row.

|  | Grp1 |  |  |  | Grp2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| R1 | \| 1 | 2 | 3 | 41 | 15 | 6 | 7 | 81 |
| R2 | \| 1 | 2 | 3 | 41 | 15 | 6 | 8 | 71 |
| R3 | \| 1 | 2 | 3 | 41 | 15 | 7 | 6 | 81 |
| R4 | \| 1 | 2 | 3 | 41 | 15 | 7 | 8 | 61 |
| R5 | \| 1 | 2 | 3 | 41 | 15 | 8 | 6 | 71 |
| R6 | \| 1 | 2 | 3 | 41 | 15 | 8 | 7 | 61 |

- Note that the permutations above are equivalent partitions of 2 groups of size 4 as only the last four elements are permuted. If we look at at the $25^{t h}$ lexicographical permutation, we observe our second distinct partition.

|  | Grp1 |  |  |  | Grp2 |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| R24 | I1 | 2 | 3 | $4 \mid$ | $\mid 8$ | 7 | 6 | $5 \mid$ |
| R25 | I 1 | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{5 \|}$ | $\mid \mathbf{4}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8} \mid$ |
| R26 | \|1 | 2 | 3 | $5 \mid$ | $\mid 4$ | 6 | 8 | $7 \mid$ |
| R27 | $\mid 1$ | 2 | 3 | $5 \mid$ | $\mid 4$ | 7 | 6 | $8 \mid$ |
| R28 | $\mid 1$ | 2 | 3 | $5 \mid$ | $\mid 4$ | 7 | 8 | $6 \mid$ |

- Continuing on, we will reach the $3,457^{\text {th }}$ lexicographical permutation, which represents the last result:

|  | Grp1 |  |  |  |  | Grp2 |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| R3454 | $\mid 1$ | 6 | 7 | $5 \mid$ | 18 | 3 | 4 | $2 \mid$ |
| R3455 | $\mid 1$ | 6 | 7 | $5 \mid$ | 18 | 4 | 2 | $3 \mid$ |
| R3456 | $\mid 1$ | 6 | 7 | $5 \mid$ | 18 | 4 | 3 | $2 \mid$ |
| R3457 | $\mid 1$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8} \mid$ | $\mid 2$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5} \mid$ |
| R3458 | $\mid 1$ | 6 | 7 | $8 \mid$ | $\mid 2$ | 3 | 5 | $4 \mid$ |

- For this small example, the method above will not be that computationally expensive. In fact, there are only 35 total partitions of $1: 8$ into 2 groups of size 4 out of a possible factorial (8) $=40320$ permutations. However, just doubling the size of the vector will make this approach infeasible as there are over 10 trillion permutations of $1: 16$.
- The algorithm in comboGroups avoids these duplicate partitions of groups by utilizing an efficient algorithm analogous to the std::next_permutation found in the standard algorithm library in $\mathrm{C}++$.


## Value

By default, a matrix is returned with column names corresponding to the associated group. If retType = "3Darray", a 3D array is returned.

## Note

The maximum number of partitions of groups that can be generated at one time is $2^{31}-1$. Utilizing lower and upper makes it possible to generate additional combinations/permutations.

## Author(s)

Joseph Wood

## Examples

```
## return a matrix
comboGroups(8, 2)
```

```
## or a 3 dimensional array
temp = comboGroups(8, 2, "3Darray")
## view the first partition
temp[1, , ]
```

    comboGroupsCount Number of Partitions of a Vector into Groups of Equal Size
    
## Description

Calculate the number of partitions of a vector into groups of equal size. See the related integer sequences A025035-A025042 at OEIS (E.g. A025036 for Number of partitions of 1, 2, ..., 4n into sets of size 4.)

## Usage <br> comboGroupsCount (v, numGroups)

## Arguments

$v \quad$ Source vector. If $v$ is a positive integer, it will be converted to the sequence $1: v$. If $v$ is a negative integer, it will be converted to the sequence $v:-1$. All atomic types are supported (See is.atomic).
numGroups An Integer. The number of groups that the vector will be partitioned into. Must divide the length of $v$ (if $v$ is a vector) or $v$ (if $v$ is a scalar).

## Value

A numerical value representing the total number of partitions of groups of equal size.

## Note

When the number of results exceeds $2^{53}-1$, a number of class bigz is returned.

## Author(s)

Joseph Wood

## References

OEIS Integer Sequence A025036

## Examples

comboGroupsCount (16, 4)

## Description

- Generate a specific (lexicographically) or random sample of partitions of groups of equal size.
- Produce results in parallel using the Parallel or nThreads arguments.
- GMP support allows for exploration where the number of results is large.


## Usage

```
comboGroupsSample(v, numGroups, retType = "matrix", n = NULL,
    sampleVec = NULL, seed = NULL, Parallel = FALSE,
    nThreads = NULL, namedSample = FALSE)
```


## Arguments

\(\left.$$
\begin{array}{ll}v & \begin{array}{l}\text { Source vector. If } v \text { is a positive integer, it will be converted to the sequence } 1: v . \\
\text { If } v \text { is a negative integer, it will be converted to the sequence } v:-1 \text {. All atomic } \\
\text { types are supported (See is. atomic). }\end{array} \\
\text { numGroups } & \begin{array}{l}\text { An Integer. The number of groups that the vector will be partitioned into. Must } \\
\text { divide the length of } v \text { (if } v \text { is a vector) or } v \text { (if } v \text { is a scalar). }\end{array} \\
\text { retType } & \begin{array}{l}\text { A string, "3Darray" or "matrix", that determines the shape of the output. The } \\
\text { default is "matrix". }\end{array} \\
\text { number of results to return. The default is NULL. }\end{array}
$$ \quad \begin{array}{l}A vector of numbers representing the lexicographical partition of groups to re- <br>
sampleVec <br>

turn. Accepts vectors of class bigz as well as vectors of characters\end{array}\right\}\)| Random seed initialization. The default is NULL. N.B. If the gmp library is |
| :--- |
| needed, this parameter must be set in order to have reproducible results (E.g |
| set. seed() has no effect in these cases). |

## Details

These algorithms rely on efficiently generating the $n^{\text {th }}$ lexicographical partition of groups of equal size.

## Value

By default, a matrix is returned with column names corresponding to the associated group. If retType = "3Darray", a 3D array is returned.

## Author(s)

Joseph Wood

## References

Lexicographical order

## Examples

```
## generate 10 random partitions of groups
comboGroupsSample(10, 2, n = 10, seed = 123)
## using sampleVec to generate specific results
comboGroupsSample(15, 5, sampleVec = c(1, 100, 1e3, 1e6))
all.equal(comboGroupsSample(10, 5,
            sampleVec = 1:comboGroupsCount(10, 5)),
    comboGroups(10, 5))
## Examples with enormous number of total results
num = comboGroupsCount (100, 20)
gmp::log2.bigz(num)
## [1] 325.5498
first = gmp::urand.bigz(n = 1, size = 325, seed = 123)
mySamp = do.call(c, lapply(0:10, function(x) gmp::add.bigz(first, x)))
class(mySamp)
## [1] "bigz"
## using the sampling function
cbgSamp = comboGroupsSample(100, 20, sampleVec = mySamp)
## using the standard function
cbgGeneral = comboGroups(100, 20,
                                    lower = first,
                                    upper = gmp::add.bigz(first, 10))
identical(cbgSamp, cbgGeneral)
## [1] TRUE
## Not run:
```

```
## Using Parallel
system.time(comboGroupsSample(1000, 20, n = 80, seed = 10, Parallel = TRUE))
## End(Not run)
```

comboIter Combination and Permutation Iterator

## Description

- Returns an iterator for iterating over combinations or permutations of a vector with or without constraints.
- Supports random access via the [[ method.
- GMP support allows for exploration of combinations/permutations of vectors with many elements.
- The output is in lexicographical order for the next methods and reverse lexicographical order for the prev methods.


## Usage

```
comboIter \((\mathrm{v}, \mathrm{m}=\mathrm{NULL}\), repetition \(=\) FALSE, freqs \(=\) NULL,
            constraintFun = NULL, comparisonFun = NULL,
            limitConstraints = NULL, keepResults = NULL,
            FUN = NULL, Parallel = FALSE, nThreads = NULL,
            tolerance \(=\) NULL, FUN.VALUE \(=\) NULL)
permuteIter (v, m = NULL, repetition = FALSE, freqs = NULL,
            constraintFun = NULL, comparisonFun = NULL,
            limitConstraints = NULL, keepResults = NULL,
            FUN = NULL, Parallel = FALSE, nThreads = NULL,
            tolerance \(=\) NULL, FUN. VALUE \(=\) NULL)
```


## Arguments

V
m
repetition
freqs A vector of frequencies used for producing all combinations/permutations of a multiset of $v$. Each element of freqs represents how many times each element of the source vector, $v$, is repeated. It is analogous to the times argument in rep. The default value is NULL.

| constraintFun | Function to be applied to the elements of $v$ that should be passed as a string (e.g. constraintFun = "sum"). The possible constraint functions are: "sum", "prod", "mean", "max", \& "min". The default is NULL, meaning no function is applied. |
| :---: | :---: |
| comparisonFun | Comparison operator that will be used to compare limitConstraints with the result of constraintFun applied to $v$. It should be passed as a string or a vector of two strings (e.g. comparisonFun = "<=" or comparisonFun = c (">", "<")). The possible comparison operators are: "<", ">", "<=", ">=", "==". The default is NULL. |
|  | When comparisonFun is a vector of two comparison strings, e.g comparisonFun $=\mathrm{c}$ (comp1, comp2), and limitConstraints is a vector of two numerical values, e.g limitConstraints $=c(x 1, x 2)$, the combinations/permutations will be filtered in one of the following two ways: |
|  | 1. When comp1 is one of the 'greater-than' operators (i.e. " $>=$ " or " $>$ "), comp2 is one of the 'less-than' operators (i.e. " $<=$ " or "<"), and $\times 1<x 2$, the combinations/permutations that are returned will have a value (after constraintFun has been applied) between $\times 1$ and $\times 2$. |
|  | 2. When comp1 and comp2 are defined as in $\# 1$ and $x 1>x 2$, the combinations/permutations that are returned will have a value outside the range of $x 1$ and $\times 2$. See the examples below. |
|  | In other words, the first comparison operator is applied to the first limit and the second operator is applied to the second limit. |
| limitConstraints |  |
|  | This is the value(s) that will be used for comparison. Can be passed as a single value or a vector of two numerical values. The default is NULL. See the definition of comparisonFun as well as the examples below for more information. |
| keepResults | A logical flag indicating if the result of constraintFun applied to $v$ should be displayed; if TRUE, an additional column of results will be added to the resulting matrix. The default is FALSE. If user is only applying constraintFun, keepResults will default to TRUE. |
| FUN | Function to be applied to each combination/permutation. The default is NULL. |
| Parallel | Logical value indicating whether combinations/permutations should be generated in parallel using $n-1$ threads, where $n$ is the maximum number of threads. The default is FALSE. If nThreads is not NULL, it will be given preference (e.g. if user has 8 threads with Parallel $=$ TRUE and $n$ Threads $=4$, only 4 threads will be spawned). If your system is single-threaded, the arguments Parallel and $n$ Threads are ignored. |
| nThreads | Specific number of threads to be used. The default is NULL. See Parallel. |
| tolerance | A numeric value greater than or equal to zero. This parameter is utilized when a constraint is applied on a numeric vector. The default value is 0 when it can be determined that whole values are being utilized, otherwise it is sqrt (.Machine\$double.eps) which is approximately $1.5 e-8$. N.B. If the input vector is of type integer, this parameter will be ignored and strict equality will be enforced. |
| FUN. VALUE | A template for the return value from FUN. See 'Details' of vapply for more information. |

## Details

Once you initialize a new iterator, the following methods are available via @ (e.g. a@nextIter()) or $\$(e . g$. a\$nextIter ()). The preferred practice is to use @ as it is much more efficient (See examples below). Also note that not all of the methods below are available in all cases. See Combo-class, Constraints-class, and Partitions-class:
nextIter Retrieve the next lexicographical result
nextNIter Pass an integer $n$ to retrieve the next $n$ lexicographical results
nextRemaining Retrieve all remaining lexicographical results
currIter Returns the current iteration
prevIter Retrieve the previous lexicographical result (the next reverse lexicographical result)
prevNIter Pass an integer $n$ to retrieve the previous $n$ lexicographical results (the next $n$ reverse lexicographical results)
prevRemaining Retrieve all remaining reverse lexicographical results
startOver Resets the iterator
sourceVector View the source vector
summary Returns a list of summary information about the iterator
front Retrieve the first lexicographical result
back Retrieve the last lexicographical result
[ [ Random access method. Pass a single value or a vector of valid indices. If a single value is passed, the internal index of the iterator will be updated, however if a vector is passed the internal state will not change. GMP support allows for flexible indexing.

## Value

- If nextIter or prevIter is called, a vector is returned
- Otherwise, a matrix with $m$ or $m+1$ columns, depending on the value of keepResults
- If FUN is utilized, FUN.VALUE = NULL, and either nextIter or prevIter is called, the result will be determined by FUN, otherwise a list is returned.
- When both FUN and FUN. VALUE are not NULL, the return is modeled after the return of vapply. See the 'Value' section of vapply.


## Note

- Parallel and $n$ Threads will be ignored in the following cases:
- When the output is constrained (except for most partitions cases)
- If the class of the vector passed is character, raw, and complex (N.B. Rcpp: :CharacterMatrix is not thread safe). Alternatively, you can generate an indexing matrix in parallel.
- If FUN is utilized.
- If either constraintFun, comparisonFun or limitConstraints is NULL -or- if the class of the vector passed is logical, character, raw, factor, or complex, the constraint check will not be carried out. This is equivalent to simply finding all combinations/permutations of $v$ choose $m$.
- The maximum number of combinations/permutations that can be generated at one time is $2^{31}-1$.
- Factor vectors are accepted. Class and level attributes are preserved except when FUN is used.
- Lexicographical ordering isn't guaranteed for permutations if the output is constrained.
- FUN will be ignored if the constraint check is satisfied.


## Author(s)

Joseph Wood

## References

- Lexicographical Order
- Reverse Lexicographical Order

See Also
comboGeneral, permuteGeneral

## Examples

```
## Typical usage
a = permuteIter(unique(state.region))
a@nextIter()
a@nextNIter(3)
a@front()
a@nextRemaining()
a@prevIter()
a@prevNIter(15)
a@summary()
a@back()
a@prevRemaining()
a[[5]]
a@summary()
a[[c(1, 17, 3)]]
a@summary()
## See examples for comboGeneral where lower and upper are used
set.seed(1009)
mySamp = sort(rnorm(75, 997, 23))
b = comboIter(mySamp, 7,
                            constraintFun = "sum",
                            comparisonFun = ">",
                            limitConstraints = 7200)
b@nextIter()
b@nextNIter(3)
b@summary()
b@currIter()
## Not run:
```

```
## We don't have random access or previous methods
b@back()
## Error: no slot of name "back" for this object of class "Constraints"
b@prevIter()
## Error: no slot of name "prevIter" for this object of class "Constraints"
## End(Not run)
```

comboRank

Rank Combinations and Permutations

## Description

- Generate the rank (lexicographically) of combinations/permutations. These functions are the complement to comboSample and permuteSample. See the examples below.
- GMP support allows for exploration of combinations/permutations of vectors with many elements.


## Usage

comboRank(..., v, repetition = FALSE, freqs = NULL)
permuteRank(..., v, repetition = FALSE, freqs $=$ NULL)

## Arguments

... vectors or matrices to be ranked.
$v \quad$ Source vector. If $v$ is a positive integer, it will be converted to the sequence $1: v$. If $v$ is a negative integer, it will be converted to the sequence $v:-1$. All atomic types are supported (See is.atomic).
repetition Logical value indicating whether combinations/permutations should be with or without repetition. The default is FALSE.
freqs A vector of frequencies used for producing all combinations/permutations of a multiset of $v$. Each element of freqs represents how many times each element of the source vector, $v$, is repeated. It is analogous to the times argument in rep. The default value is NULL.

## Details

These algorithms rely on efficiently ranking the $n^{\text {th }}$ lexicographical combination/permutation.

## Value

A vector of class integer, numeric, or bigz determined by the total number of combinations/permutations

## Note

$\checkmark$ must be supplied.

## Author(s)

Joseph Wood

## References

Lexicographical order ranking/unranking

## See Also

comboSample, permuteSample

## Examples

```
mySamp = comboSample(30, 8, TRUE, n = 5, seed = 10, namedSample = TRUE)
myRank = comboRank(mySamp, v = 30, repetition = TRUE)
all.equal(as.integer(rownames(mySamp)), myRank)
```

```
comboSample Sample Combinations and Permutations
```


## Description

- Generate a specific (lexicographically) or random sample of combinations/permutations.
- Produce results in parallel using the Parallel or nThreads arguments.
- GMP support allows for exploration of combinations/permutations of vectors with many elements.


## Usage

```
comboSample(v, m = NULL, repetition = FALSE, freqs = NULL, n = NULL,
            sampleVec = NULL, seed = NULL, FUN = NULL, Parallel = FALSE,
            nThreads = NULL, namedSample = FALSE, FUN.VALUE = NULL)
permuteSample(v, m = NULL, repetition = FALSE, freqs = NULL, n = NULL,
                sampleVec = NULL, seed = NULL, FUN = NULL, Parallel = FALSE,
                nThreads = NULL, namedSample = FALSE, FUN.VALUE = NULL)
```


## Arguments

v
m
repetition Logical value indicating whether combinations/permutations should be with or without repetition. The default is FALSE.

| freqs | A vector of frequencies used for producing all combinations/permutations of a multiset of $v$. Each element of freqs represents how many times each element of the source vector, $v$, is repeated. It is analogous to the times argument in rep. The default value is NULL. |
| :---: | :---: |
| n | Number of combinations/permutations to return. The default is NULL. |
| sampleVec | A vector of indices representing the lexicographical combination/permutations to return. Accepts whole numbers as well as vectors of class bigz as well as vectors of characters |
| seed | Random seed initialization. The default is NULL. N.B. If the gmp library is needed, this parameter must be set in order to have reproducible results (E.g set. seed () has no effect in these cases). |
| FUN | Function to be applied to each combination/permutation. The default is NULL. |
| Parallel | Logical value indicating whether combinations/permutations should be generated in parallel. The default is FALSE. If TRUE and nThreads = NULL, the number of threads used is equal to the minimum of one minus the number of threads available on your system and the number of results requested (e.g. if user has 16 threads and only needs 5 results, 5 threads will be used (i.e. $\min (16-1,5)=$ $5)$ ). If $n$ Threads is not NULL, it will be given preference (e.g. if user has 8 threads with Parallel $=$ TRUE and $n$ Threads $=4$, only 4 threads will be spawned). If your system is single-threaded, the arguments Parallel and nThreads are ignored. |
| nThreads | Specific number of threads to be used. The default is NULL. See Parallel. |
| namedSample | Logical flag. If TRUE, rownames corresponding to the lexicographical combination/permutation, will be added to the returned matrix. The default is FALSE. |
| FUN. VALUE | A template for the return value from FUN. See 'Details' of vapply for more information. |

## Details

These algorithms rely on efficiently generating the $n^{\text {th }}$ lexicographical combination/permutation. This is the process of unranking.

## Value

- In general, a matrix with $m$ or $m+1$ columns, depending on the value of keepResults
- If FUN is utilized and FUN. VALUE = NULL, a list is returned
- When both FUN and FUN. VALUE are not NULL, the return is modeled after the return of vapply. See the 'Value' section of vapply.


## Note

- Parallel and nThreads will be ignored in the following cases:
- If the class of the vector passed is character (N.B. Rcpp: CharacterMatrix is not thread safe). Alternatively, you can generate an indexing matrix in parallel.
- If FUN is utilized.
- n and sampleVec cannot both be NULL.
- Factor vectors are accepted. Class and level attributes are preserved except when FUN is used.


## Author(s)

Joseph Wood

## References

Lexicographical order

## See Also

comboRank, permuteRank

## Examples

```
## generate 10 random combinations
comboSample(30, 8, TRUE, n = 5, seed = 10)
## using sampleVec to generate specific permutations
permuteSample(15, 10, freqs = c(1,2,2,1,2,2,1,2,1,2,2,1,2,1,1),
            sampleVec = c(1, 10^2, 10^5, 10^8, 10^11))
all.equal(comboSample(10, 5,
                sampleVec = 1:comboCount(10, 5)),
    comboGeneral(10, 5))
## Examples with enormous number of total permutations
num = permuteCount(10000, 20)
gmp::log2.bigz(num)
## [1] 265.7268
first = gmp::urand.bigz(n = 1, size = 265, seed = 123)
mySamp = do.call(c, lapply(0:10, function(x) gmp::add.bigz(first, x)))
class(mySamp)
## [1] "bigz"
## using permuteSample
pSamp = permuteSample(10000, 20, sampleVec = mySamp)
## using permuteGeneral
pGeneral = permuteGeneral(10000, 20,
                                    lower = first,
                                    upper = gmp::add.bigz(first, 10))
identical(pSamp, pGeneral)
## [1] TRUE
## Using nThreads
permPar = permuteSample(10000, 50, n = 8, seed = 10, nThreads = 2)
## Using FUN
permuteSample(10000, 50, n = 4, seed = 10, FUN = sd)
```

```
## Not run:
## Using Parallel
permuteSample(10000, 50, n = 80, seed = 10, Parallel = TRUE)
## End(Not run)
```

```
Constraints-class S4-class for Exposing C++ Constraints Class
```


## Description

The Constraints class is an S4-class that exposes C++ classes that provide access to iterators and other useful methods.

## Slots

nextIter Retrieve the next lexicographical result
nextNIter Pass an integer $n$ to retrieve the next $n$ lexicographical results
nextRemaining Retrieve all remaining lexicographical results
currIter Returns the current iteration
startOver Resets the iterator
sourceVector View the source vector
summary Returns a list of summary information about the iterator

## Author(s)

Joseph Wood

## See Also

```
Combo-class, Partitions-class
```


## Examples

```
    showClass("Constraints")
```

```
    divisorsRcpp Vectorized Factorization(Complete)
```


## Description

Function for generating the complete factorization for a vector of numbers.

## Usage

divisorsRcpp(v, namedList = FALSE, nThreads = NULL)

## Arguments

v
namedList Logical flag. If TRUE and the length $(v)>1$, a named list is returned. The default is FALSE.
nThreads $\quad$ Specific number of threads to be used. The default is NULL.

## Details

Efficient algorithm that builds on primeFactorize to generate the complete factorization of many numbers.

## Value

- Returns an unnamed vector if length(v) == 1 regardless of the value of namedList. If $v<$ $2^{31}$, the class of the returned vector will be integer, otherwise the class will be numeric.
- If length $(v)>1$, a named/unnamed list of vectors will be returned. If max (bound1, bound2) $<2^{31}$, the class of each vector will be integer, otherwise the class will be numeric.


## Note

The maximum value for each element in $v$ is $2^{53}-1$.

## Author(s)

Joseph Wood

## References

- Divisor
- 53-bit significand precision

See Also
divisors, primeFactorize

## Examples

```
## Get the complete factorization of a single number
divisorsRcpp(10^8)
## Or get the complete factorization of many numbers
set.seed(29)
myVec <- sample(-1000000:1000000, 1000)
system.time(myFacs <- divisorsRcpp(myVec))
## Return named list
myFacsWithNames <- divisorsRcpp(myVec, namedList = TRUE)
## Using nThreads
system.time(divisorsRcpp(myVec, nThreads = 2))
```

divisorsSieve Generate Complete Factorization for Numbers in a Range

## Description

Sieve that generates the complete factorization of all numbers between bound1 and bound2 (if supplied) or all numbers up to bound1.

## Usage

divisorsSieve(bound1, bound2 = NULL, namedList = FALSE, nThreads = NULL)

## Arguments

bound1 Positive integer or numeric value.
bound2 Positive integer or numeric value.
namedList Logical flag. If TRUE, a named list is returned. The default is FALSE.
nThreads Specific number of threads to be used. The default is NULL.

## Details

This function is useful when many complete factorizations are needed. Instead of generating the complete factorization on the fly, one can reference the indices/names of the generated list.
This algorithm benefits greatly from the fast integer division library 'libdivide'. The following is from http://libdivide.com/:

- "libdivide allows you to replace expensive integer divides with comparatively cheap multiplication and bitshifts. Compilers usually do this, but only when the divisor is known at compile time. libdivide allows you to take advantage of it at runtime. The result is that integer division can become faster - a lot faster."


## Value

Returns a named/unnamed list of integer vectors if $\max$ (bound1, bound2) $<2^{31}$, or a list of numeric vectors otherwise.

## Note

The maximum value for either of the bounds is $2^{53}-1$.

## Author(s)

Joseph Wood

## References

- Divisor
- ridiculousfish (author of libdivide)
- github.com/ridiculousfish/libdivide
- 53-bit significand precision


## See Also

divisorsRcpp, divisors, primeFactorizeSieve

## Examples

```
## Generate some random data
set.seed(33550336)
mySamp <- sample(10^5, 5*10^4)
## Generate complete factorizations up
## to 10^5 (max element from mySamp)
system.time(allFacs <- divisorsSieve(10^5))
## Use generated complete factorization for further
## analysis by accessing the index of allFacs
for (s in mySamp) {
    myFac <- allFacs[[s]]
    ## Continue algorithm
}
## Generating complete factorizations over
## a range is efficient as well
system.time(divisorsSieve(10^12, 10^12 + 10^5))
## Use nThreads for improved efficiency
system.time(divisorsSieve(10^12, 10^12 + 10^5, nThreads = 2))
## Set 'namedList' to TRUE to return a named list
divisorsSieve(27, 30, namedList = TRUE)
```

```
## Using nThreads
```

system.time(divisorsSieve(1e5, 2e5, nThreads = 2))

```
eulerPhiSieve Apply Euler's Phi Function to Every Element in a Range
```


## Description

Sieve that generates the number of coprime elements for every number between bound 1 and bound 2 (if supplied) or all numbers up to bound1. This is equivalent to applying Euler's phi function (often written as $\phi(x)$ ) to every number in a given range.

## Usage

eulerPhiSieve(bound1, bound2 $=$ NULL, namedVector $=$ FALSE, $n$ Threads $=$ NULL)

## Arguments

bound1 Positive integer or numeric value.
bound2 Positive integer or numeric value.
namedVector Logical flag. If TRUE, a named vector is returned. The default is FALSE.
nThreads Specific number of threads to be used. The default is NULL.

## Details

For the simple case (i.e. when bound $2=$ NULL), this algorithm first generates all primes up to $n$ via the sieve of Eratosthenes. We use these primes to sieve over the sequence $1: n$, dividing each value by $p$, creating a temporary value that will be subtracted from the original value at each index (i.e. equivalent to multiply each index by $(1-1 / p)$ but more efficient as we don't have to deal with floating point numbers). The case when is.null(bound2) = FALSE is more complicated but the basic ideas still hold.
This function is very useful when you need to calculate Euler's phi function for many numbers in a range as performing this calculation on the fly can be computationally expensive.
This algorithm benefits greatly from the fast integer division library 'libdivide'. The following is from http://libdivide.com/:

- "libdivide allows you to replace expensive integer divides with comparatively cheap multiplication and bitshifts. Compilers usually do this, but only when the divisor is known at compile time. libdivide allows you to take advantage of it at runtime. The result is that integer division can become faster - a lot faster."


## Value

Returns a named/unnamed integer vector if $\max$ (bound1, bound2) $<2^{31}$, or a numeric vector otherwise.

## Note

The maximum allowed value is $2^{53}-1$.

## Author(s)

Joseph Wood

## References

- Euler's totient function
- ridiculousfish (author of libdivide)
- github.com/ridiculousfish/libdivide
- 53-bit significand precision


## See Also

eulersPhi

## Examples

```
## Generate some random data
set.seed(496)
mySamp <- sample(10^6, 5*10^5)
## Generate number of coprime elements for many numbers
system.time(myPhis <- eulerPhiSieve(10^6))
## Now use result in algorithm
for (s in mySamp) {
    sPhi <- myPhis[s]
    ## Continue algorithm
}
## See https://projecteuler.net
system.time(which.max((1:10^6)/eulerPhiSieve(10^6)))
## Generating number of coprime elements
## for every number in a range is no problem
system.time(myPhiRange <- eulerPhiSieve(10^13, 10^13 + 10^6))
## Returning a named vector
eulerPhiSieve(10, 20, namedVector = TRUE)
eulerPhiSieve(10, namedVector = TRUE)
## Using nThreads
system.time(eulerPhiSieve(1e5, 2e5, nThreads = 2))
```

isPrimeRcpp Vectorized Primality Test

## Description

Implementation of the Miller-Rabin primality test. Based on the "mp_prime_p" function from the "factorize.c" source file found in the gmp library: https://gmplib.org.

## Usage

isPrimeRcpp(v, namedVector $=$ FALSE, $n$ Threads $=$ NULL)

## Arguments

$\checkmark \quad$ Vector of integers or numeric values.
namedVector Logical flag. If TRUE, a named vector is returned. The default is FALSE.
nThreads Specific number of threads to be used. The default is NULL.

## Details

The Miller-Rabin primality test is a probabilistic algorithm that makes heavy use of modular exponentiation. At the heart of modular exponentiation is the ability to accurately obtain the remainder of the product of two numbers $(\bmod p)$.
With the gmp library, producing accurate calculations for problems like this is trivial because of the nature of the multiple precision data type. However, standard C++ does not afford this luxury and simply relying on a strict translation would have limited this algorithm to numbers less than $\sqrt{2}^{63}-1$ (N.B. We are taking advantage of the signed 64-bit fixed width integer from the stdint library in $\mathrm{C}++$. If we were confined to base $R$, the limit would have been $\sqrt{2}^{53}-1$ ). RcppAlgos::isPrimeRcpp gets around this limitation with a divide and conquer approach taking advantage of properties of arithmetic.

The problem we are trying to solve can be summarized as follows:

$$
\left(x_{1} * x_{2}\right) \quad(\bmod p)
$$

Now, we rewrite $x_{2}$ as $x_{2}=y_{1}+y_{2}+\ldots+y_{n}$, so that we obtain:

$$
\left(x_{1} * y_{1}\right) \quad(\bmod p)+\left(x_{1} * y_{2}\right) \quad(\bmod p)+\ldots+\left(x_{1} * y_{n}\right) \quad(\bmod p)
$$

Where each product $\left(x_{1} * y_{j}\right)$ for $j<=n$ is smaller than the original $x_{1} * x_{2}$. With this approach, we are now capable of handling much larger numbers. Many details have been omitted for clarity.
For a more in depth examination of this topic see Accurate Modular Arithmetic with Double Precision.

## Value

Returns a named/unnamed logical vector. If an index is TRUE, the number at that index is prime, otherwise the number is composite.

## Note

The maximum value for each element in $v$ is $2^{53}-1$.

## References

- THE MILLER-RABIN TEST
- Conrad, Keith. "THE MILLER-RABIN TEST." http://www.math.uconn.edu/~kconrad/blurbs/ugradnumthy/miller
- 53-bit significand precision


## See Also

primeFactorize, isprime, isPrime

## Examples

```
## check the primality of a single number
isPrimeRcpp(100)
## check the primality of every number in a vector
isPrimeRcpp(1:100)
set.seed(42)
mySamp <- sample(10^13, 10)
## return named vector for easy identification
isPrimeRcpp(mySamp, namedVector = TRUE)
## Using nThreads
system.time(isPrimeRcpp(mySamp, nThreads = 2))
```

numDivisorSieve Apply Divisor Function to Every Element in a Range

## Description

Sieve that generates the number of divisors for every number between bound 1 and bound 2 (if supplied) or all numbers up to bound1. This is equivalent to applying the divisor function (often written as $\sigma(x)$ ) to every number in a given range.

## Usage

numDivisorSieve(bound1, bound2 = NULL, namedVector = FALSE, nThreads = NULL)

## Arguments

bound1 Positive integer or numeric value.
bound2 Positive integer or numeric value.
namedVector Logical flag. If TRUE, a named vector is returned. The default is FALSE.
nThreads $\quad$ Specific number of threads to be used. The default is NULL.

## Details

Simple and efficient sieve that calculates the number of divisors for every number in a given range. This function is very useful when you need to calculate the number of divisors for many numbers.
This algorithm benefits greatly from the fast integer division library 'libdivide'. The following is from http://libdivide.com/:

- "libdivide allows you to replace expensive integer divides with comparatively cheap multiplication and bitshifts. Compilers usually do this, but only when the divisor is known at compile time. libdivide allows you to take advantage of it at runtime. The result is that integer division can become faster - a lot faster."


## Value

Returns a named/unnamed integer vector

## Note

The maximum allowed value is $2^{53}-1$.

## Author(s)

Joseph Wood

## References

- Divisor function
- ridiculousfish (author of libdivide)
- github.com/ridiculousfish/libdivide
- 53-bit significand precision


## Examples

```
## Generate some random data
set.seed(8128)
mySamp <- sample(10^6, 5*10^5)
## Generate number of divisors for
## every number less than a million
system.time(mySigmas <- numDivisorSieve(10^6))
## Now use result in algorithm
```

```
for (s in mySamp) {
    sSig <- mySigmas[s]
    ## Continue algorithm
}
## Generating number of divisors for every
## number in a range is no problem
system.time(sigmaRange <- numDivisorSieve(10^13, 10^13 + 10^6))
## Returning a named vector
numDivisorSieve(10, 20, namedVector = TRUE)
numDivisorSieve(10, namedVector = TRUE)
## Using nThreads
system.time(numDivisorSieve(1e5, 2e5, nThreads = 2))
```

Partitions-class S4-class for Exposing C++ Partitions Class

## Description

The Partitions class is an S4-class that exposes C++ classes that provide access to iterators and other useful methods.

## Slots

nextIter Retrieve the next lexicographical result
nextNIter Pass an integer $n$ to retrieve the next $n$ lexicographical results
nextRemaining Retrieve all remaining lexicographical results
currIter Returns the current iteration
startOver Resets the iterator
sourceVector View the source vector
summary Returns a list of summary information about the iterator
front Retrieve the first lexicographical result
back Retrieve the last lexicographical result
randomAccess Random access method. Pass a single value or a vector of valid indices. If a single value is passed, the internal index of the iterator will be updated, however if a vector is passed the internal state will not change. GMP support allows for flexible indexing.

## Author(s)

Joseph Wood

See Also
Combo-class, Constraints-class

## Examples

```
showClass("Partitions")
```

```
partitionsCount Number of Partitions/Compositions
```


## Description

Calculate the number of partitions/compositions of a vector chosen $m$ at a time with or without replacement. Additionally, these functions can calculate the number of partitions of multisets.

## Usage

```
partitionsCount(v, m = NULL, repetition = FALSE,
    freqs = NULL, target = NULL)
compositionsCount(v, m = NULL, repetition = FALSE,
    freqs = NULL, target = NULL, weak = FALSE)
```


## Arguments

$v \quad$ Source vector. If $v$ is a positive integer, it will be converted to the sequence $1: v$. If $v$ is a negative integer, it will be converted to the sequence $v:-1$. Only integer and numeric vectors are accepted.
$\mathrm{m} \quad$ Width of the partition. If $m=$ NULL, the length will be determined by the partitioning case (e.g. When we are generating distinct partitions of $n$, the width will be equal to the smallest $m$ such that sum( $1: m$ ) $>=n$ ).
repetition Logical value indicating whether partitions/compositions should be with or without repetition. The default is FALSE.
freqs A vector of frequencies used for producing all partitions of a multiset of v. Each element of freqs represents how many times each element of the source vector, $v$, is repeated. It is analogous to the times argument in rep. The default value is NULL.
target Number to be partitioned. If NULL, the max (v) will be used.
weak (Compositions only) Logical flag indicating whether to allow terms of the sequence to be zero.

## Value

A numerical value representing the total number of partitions/compositions.

## Note

When the number of results exceeds $2^{53}-1$, a number of class bigz is returned.

## See Also

```
partitionsGeneral, compositionsGeneral
```


## Examples

```
## Same interface as partitionsGeneral
partitionsCount(25, 5)
compositionsCount(25, 5, TRUE)
partitionsCount(15, 7, TRUE)
partitionsCount(25, 5, freqs = rep(2, 25))
## Return object of class 'bigz'
partitionsCount(2500, 15, TRUE)
compositionsCount(2500, 15, TRUE)
```

partitionsGeneral Generate Partitions/Compositions

## Description

- Efficient algorithms for partitioning numbers under various constraints:
- Standard (with repetition)
- Distinct
- Restricted
- Where each part has a specific multiplicity (i.e. when using freqs for multisets).
- Arbitrary target and source vector (e.g. partitionsGeneral(sample (1000, 20), 10, TRUE, target = 5000) )
- Produce results in parallel using the $n$ Threads arguments.
- Alternatively, the arguments lower and upper make it possible to generate partitions/compositions in chunks allowing for parallelization via the parallel package.
- GMP support allows for exploration of cases where the number of partitions/compositions is large.
- The output is in lexicographical order.


## Usage

```
partitionsGeneral(v, m = NULL, repetition = FALSE,
                        freqs = NULL, target = NULL, lower = NULL,
        upper = NULL, nThreads = NULL,
        tolerance = NULL)
compositionsGeneral(v, m = NULL, repetition = FALSE,
    freqs = NULL, target = NULL, weak = FALSE,
    lower = NULL, upper = NULL, nThreads = NULL,
    tolerance = NULL)
```


## Arguments

| v | Source vector. If $v$ is a positive integer, it will be converted to the sequence $1: v$. If $v$ is a negative integer, it will be converted to the sequence $v:-1$. Only integer and numeric vectors are accepted. |
| :---: | :---: |
| m | Width of the partition. If $m=$ NULL, the length will be determined by the partitioning case (e.g. When we are generating distinct partitions of $n$, the width will be equal to the smallest $m$ such that sum ( $1: m$ ) $>=n$ ). |
| repetition | Logical value indicating whether partitions/compositions should be with or without repetition. The default is FALSE. |
| freqs | A vector of frequencies used for producing all partitions of a multiset of v. Each element of freqs represents how many times each element of the source vector, $v$, is repeated. It is analogous to the times argument in rep. The default value is NULL. |
| lower | The lower bound. Partitions/compositions are generated lexicographically, thus utilizing this argument will determine which specific partition to start generating from (e.g. partitionsGeneral $(15,3$, lower $=6)$ is equivalent to partitionsGeneral (15, 3) [6: partitionsCount $(15,3)$,$] ). This argument along with upper is very$ useful for generating partitions/compositions in chunks allowing for easy parallelization. |
| upper | The upper bound. Similar to lower, however this parameter allows the user to stop generation at a specific partition (e.g. partitionsGeneral(15, 3, upper $=5)$ is equivalent to partitionsGeneral $(15,3)[1: 5]$, |
| target | Number to be partitioned. If NULL, the max (v) will be used. |
| weak | (Compositions only) Logical flag indicating whether to allow terms of the sequence to be zero. |
| nThreads | Specific number of threads to be used. The default is NULL. |
| tolerance | A numeric value greater than or equal to zero. This parameter is utilized when a constraint is applied on a numeric vector. The default value is 0 when it can be determined that whole values are being utilized, otherwise it is sqrt (.Machine\$double.eps) which is approximately $1.5 e-8$. N.B. If the input vector is of type integer, this parameter will be ignored and strict equality will be enforced. |

## Value

A matrix is returned with each row containing a vector of length $m$.

## Note

- nThreads will be ignored in the following cases (i.e. Generating the $n^{t h}$ partition in these cases are currently unavailable):
- With standard multisets. If zero is the only element with a non-trivial multiplicity, multithreading is possible (e.g. partitionsGeneral (0:100, freqs $=c(100, r e p(1,100))$, nThreads = 4)).
- If the source vector is not isomorphic to 1 : length $(v)($ e.g. $v=c(1,4,6,7,8))$.
- The maximum number of partitions/compositions that can be generated at one time is $2^{31}-1$. Utilizing lower and upper makes it possible to generate additional partitions/compositions.


## Author(s)

Joseph Wood

## References

- Lexicographical Order
- Subset Sum Problem
- Partition (number theory)
- Composition (combinatorics))


## Examples

```
partitionsGeneral(1)
partitionsGeneral(-1:0, 1)
partitionsGeneral(-1:0, 1, target = -1)
partitionsGeneral(20, 5)
partitionsGeneral(20, 5, repetition = TRUE)
partitionsGeneral(20, 5, freqs = rep(1:4, 5))
partitionsGeneral(20, 5, TRUE, target = 80)
partitionsGeneral(0:10, repetition = TRUE)
partitionsGeneral(seq(2L, 500L, 23L), 5, target = 1804)
compositionsGeneral(0:10, 5, repetition = TRUE)
set.seed(111)
partitionsGeneral(sample(1000, 20), 5, TRUE, target = 2500)
system.time(one_thread <- partitionsGeneral(80, 10, TRUE))
system.time(two_threads <- partitionsGeneral(80, 10, TRUE, nThreads = 2))
identical(one_thread, two_threads)
```

partitionsIter Partition/Composition Iterator

## Description

- Returns an iterator for iterating over partitions/compositions of a numbers.
- Supports random access via the [[ method.
- GMP support allows for exploration of cases where the number of partitions/compositions is large.
- Use the next methods to obtain results in lexicographical order.


## Usage

```
partitionsIter(v, m = NULL, repetition = FALSE,
            freqs \(=\) NULL, target \(=\) NULL,
            nThreads \(=\) NULL, tolerance \(=\) NULL)
compositionsIter(v, m = NULL, repetition = FALSE, freqs = NULL,
                    target \(=\) NULL, weak \(=\) FALSE, \(n\) Threads \(=\) NULL,
                        tolerance \(=\) NULL)
```


## Arguments

$v \quad$ Source vector. If $v$ is a positive integer, it will be converted to the sequence $1: v$. If $v$ is a negative integer, it will be converted to the sequence $v:-1$. Only integer and numeric vectors are accepted.
m
Width of the partition. If $m=$ NULL, the length will be determined by the partitioning case (e.g. When we are generating distinct partitions of $n$, the width will be equal to the smallest $m$ such that sum ( $1: m$ ) $>=n$ ).
repetition Logical value indicating whether partitions/compositions should be with or without repetition. The default is FALSE.
freqs A vector of frequencies used for producing all partitions of a multiset of $v$. Each element of freqs represents how many times each element of the source vector, $v$, is repeated. It is analogous to the times argument in rep. The default value is NULL.
target Number to be partitioned. If NULL, the $\max (v)$ will be used.
weak (Compositions only) Logical flag indicating whether to allow terms of the sequence to be zero.
nThreads $\quad$ Specific number of threads to be used. The default is NULL.
tolerance A numeric value greater than or equal to zero. This parameter is utilized when a constraint is applied on a numeric vector. The default value is 0 when it can be determined that whole values are being utilized, otherwise it is sqrt (.Machine\$double.eps) which is approximately $1.5 e-8$. N.B. If the input vector is of type integer, this parameter will be ignored and strict equality will be enforced.

## Details

Once you initialize a new iterator, the following methods are available:
nextIter Retrieve the next lexicographical result
nextNIter Pass an integer $n$ to retrieve the next $n$ lexicographical results
nextRemaining Retrieve all remaining lexicographical results
currIter Returns the current iteration
startOver Resets the iterator
sourceVector View the source vector
summary Returns a list of summary information about the iterator
front Retrieve the first lexicographical result
back Retrieve the last lexicographical result
[ [ Random access method. Pass a single value or a vector of valid indices. If a single value is passed, the internal index of the iterator will be updated, however if a vector is passed the internal state will not change. GMP support allows for flexible indexing.

## Value

- If nextIter or prevIter is called, a vector is returned
- Otherwise, a matrix with $m$ columns


## Note

- If nThreads is utilized, it will only take effect if the number of elements requested is greater than some threshold (determined internally). E.g:

```
serial <- partitionsIter(1000, 10)
multi <- partitionsIter(1000, 10, nThreads = 4)
fetch1e6 <- multi@nextNIter(1e6) ## much faster than serial@nextNIter(1e6)
fetch1e3 <- multi@nextNIter(1e3) ## only one thread used... same as serial@nextNIter(1e3)
library(microbenchmark)
microbenchmark(multi@nextNIter(1e6), serial@nextNIter(1e6))
microbenchmark(multi@nextNIter(1e3), serial@nextNIter(1e3))
```

- nThreads will be ignored in the following cases (i.e. Generating the $n^{t h}$ partition in these cases are currently unavailable):
- With standard multisets. If zero is the only element with a non-trivial multiplicity, multithreading is possible.
- If the source vector is not isomorphic to 1:length(v)
- The maximum number of partitions/compositions that can be generated at one time is $2^{31}-1$.


## Author(s)

Joseph Wood

## References

- Lexicographical Order
- Subset Sum Problem
- Partition (number theory)
- Composition (combinatorics))


## See Also

partitionsGeneral, compositionsGeneral

## Examples

```
a = partitionsIter(0:10, repetition = TRUE)
a@nextIter()
a@nextNIter(3)
a@front()
a@nextRemaining()
a@summary()
a@back()
a[[5]]
a@summary()
a[[c(1, 17, 3)]]
a@summary()
## Multisets... no random access
b = partitionsIter(40, 5, freqs = rep(1:4, 10), target = 80)
b@nextIter()
b@nextNIter(10)
b@summary()
b@nextIter()
b@currIter()
```

partitionsRank Rank Partitions/Compositions

## Description

- Generate the rank (lexicographically) of partitions/compositions. These functions are the complement to partitions/compositionsSample. See the examples below.
- GMP support allows for exploration of partitions/compositions of vectors with many elements.


## Usage

```
partitionsRank(..., v, repetition = FALSE, freqs = NULL, target = NULL)
compositionsRank(..., v, repetition = FALSE, freqs = NULL,
            target = NULL, weak = FALSE)
```


## Arguments

... vectors or matrices to be ranked.
$v \quad$ Source vector. If $v$ is a positive integer, it will be converted to the sequence $1: v$. If $v$ is a negative integer, it will be converted to the sequence $v:-1$. All atomic types are supported (See is.atomic).
repetition Logical value indicating whether partitions/compositions should be with or without repetition. The default is FALSE.

| freqs | A vector of frequencies used for producing all partitions of a multiset of $v$. Each <br> element of freqs represents how many times each element of the source vector, <br> $v$, is repeated. It is analogous to the times argument in rep. The default value <br> is NULL. |
| :--- | :--- |
| target | Number to be partitioned. If NULL, the max $(v)$ will be used. <br> weak <br> (Compositions only) Logical flag indicating whether to allow terms of the se- <br> quence to be zero. |

## Details

These algorithms rely on efficiently ranking the $n^{t h}$ lexicographical partition.
Value
A vector of class integer, numeric, or bigz determined by the total number of partitions/compositions

## Note

$\vee$ must be supplied.

## Author(s)

Joseph Wood

## References

Lexicographical order ranking/unranking

## See Also

partitionsSample, compositionsSample

## Examples

```
mySamp = partitionsSample(30, 8, TRUE, n = 5, seed = 10, namedSample = TRUE)
myRank = partitionsRank(mySamp, v = 30, repetition = TRUE)
all.equal(as.integer(rownames(mySamp)), myRank)
```

```
partitionsSample Sample Partitions/Compositions
```


## Description

- Generate a specific (lexicographically) or random sample of partitions/compositions of a number.
- Produce results in parallel using the Parallel or nThreads arguments.
- GMP support allows for exploration of cases where the number of partitions/compositions is large.


## Usage

```
partitionsSample(v, m = NULL, repetition = FALSE, freqs = NULL,
    target \(=\) NULL, \(n=\) NULL, sampleVec \(=\) NULL,
    seed \(=\) NULL, nThreads \(=\) NULL, namedSample \(=\) FALSE)
compositionsSample(v, m = NULL, repetition = FALSE, freqs = NULL,
        target \(=\) NULL, weak \(=\) FALSE, \(\mathrm{n}=\) NULL, sampleVec \(=\) NULL,
        seed \(=\) NULL, nThreads \(=\) NULL, namedSample \(=\) FALSE)
```


## Arguments

$v \quad$ Source vector. If $v$ is a positive integer, it will be converted to the sequence $1: v$. If $v$ is a negative integer, it will be converted to the sequence $v:-1$. Only integer and numeric vectors are accepted.
m
Width of the partition. If $m=$ NULL, the length will be determined by the partitioning case (e.g. When we are generating distinct partitions of $n$, the width will be equal to the smallest $m$ such that sum ( $1: m$ ) $>=n$ ).
repetition Logical value indicating whether partitions/compositions should be with or without repetition. The default is FALSE.
freqs A vector of frequencies used for producing all partitions of a multiset of $v$. Each element of freqs represents how many times each element of the source vector, v , is repeated. It is analogous to the times argument in rep. The default value is NULL.
target Number to be partitioned. If NULL, the $\max (v)$ will be used.
weak (Compositions only) Logical flag indicating whether to allow terms of the sequence to be zero.
$\mathrm{n} \quad$ Number of partitions/compositions to return. The default is NULL.
sampleVec A vector of numbers representing the lexicographical partitions/compositions to return. Accepts vectors of class bigz as well as vectors of characters
seed Random seed initialization. The default is NULL. N.B. If the gmp library is needed, this parameter must be set in order to have reproducible results ( $E . g$ set. seed () has no effect in these cases).
nThreads Specific number of threads to be used. The default is NULL.
namedSample Logical flag. If TRUE, rownames corresponding to the lexicographical partition, will be added to the returned matrix. The default is FALSE.

## Details

These algorithms rely on efficiently generating the $n^{\text {th }}$ lexicographical partition. This is the process of unranking.

## Value

A matrix is returned with each row containing a vector of length $m$.

## Note

- partitionsSample is not available for the following cases:
- With standard multisets. If zero is the only element with a non-trivial multiplicity, sampling is allowed (e.g. partitionsSample (0:100, freqs $=c(100, r e p(1,100)), n=$ 2))
- If the source vector is not isomorphic to 1 : length $(v)(e . g . v=c(1,4,6,7,8))$.
- n and sampleVec cannot both be NULL.


## Author(s)

Joseph Wood

## References

- Lexicographical order
- Partition (Number Theory)


## Examples

```
partitionsSample(100, 10, n = 5)
partitionsSample(100, 10, seed = 42, n = 5, target = 200)
## retrieve specific results (lexicographically)
partitionsCount(100, 10, TRUE, target = 500)
## [1] 175591757896
partitionsSample(100, 10, TRUE, target = 500,
    sampleVec = c(1, 1000, 175591757896))
```

    primeCount Prime Counting Function \(\pi(x)\)
    
## Description

Prime counting function for counting the prime numbers less than an integer, $n$, using Legendre's formula. It is based on the the algorithm developed by Kim Walisch found here: kimwalisch/primecount.

## Usage

primeCount(n, nThreads $=$ NULL)

## Arguments

n
nThreads

Positive number
Specific number of threads to be used. The default is NULL.

## Details

Legendre's Formula for counting the number of primes less than $n$ makes use of the inclusionexclusion principle to avoid explicitly counting every prime up to $n$. It is given by:

$$
\pi(x)=\pi(\sqrt{x})+\Phi(x, \sqrt{x})-1
$$

Where $\Phi(x, a)$ is the number of positive integers less than or equal to $x$ that are relatively prime to the first $a$ primes (i.e. not divisible by any of the first $a$ primes). It is given by the recurrence relation ( $p_{a}$ is the $a t h$ prime (e.g. $p_{4}=7$ )):

$$
\Phi(x, a)=\Phi(x, a-1)+\Phi\left(x / p_{a}, a-1\right)
$$

This algorithm implements five modifications developed by Kim Walisch for calculating $\Phi(x, a)$ efficiently.

1. Cache results of $\Phi(x, a)$
2. Calculate $\Phi(x, a)$ using $\Phi(x, a)=(x / p p) * \phi(p p)+\Phi(x \bmod p p, a)$ if $a<=6$

- $p p=2 * 3 * \ldots *$ prime[a]
- $\phi(p p)=(2-1) *(3-1) * \ldots *($ prime[a] -1$)$ (i.e. Euler's totient function)

3. Calculate $\Phi(x, a)$ using $\pi(x)$ lookup table
4. Calculate all $\Phi(x, a)=1$ upfront
5. Stop recursion at 6 if $\sqrt{x}>=13$ or $\pi(\sqrt{x})$ instead of 1

## Value

Whole number representing the number of prime numbers less than or equal to $n$.

## Note

The maximum value of $n$ is $2^{53}-1$

## Author(s)

Joseph Wood

## References

- Computing $\pi(x)$ : the combinatorial method
- Tomás Oliveira e Silva, Computing pi(x): the combinatorial method, Revista do DETUA, vol. 4, no. 6, March 2006, p. 761. http://sweet.ua.pt/tos/bib/5.4.pdf
- 53-bit significand precision


## See Also

primeSieve

## Examples

```
## Get the number of primes less than a billion
primeCount(10^9)
## Using nThreads
system.time(primeCount(10^10, nThreads = 2))
```

```
primeFactorize Vectorized Prime Factorization
```


## Description

Implementation of Pollard's rho algorithm for generating the prime factorization. The algorithm is based on the "factorize.c" source file from the gmp library found here $\mathrm{https}: / / \mathrm{gmplib} . o r g$.

## Usage

primeFactorize(v, namedList $=$ FALSE, nThreads $=$ NULL)

## Arguments

v
namedList
nThreads Specific number of threads to be used. The default is NULL.

## Details

As noted in the Description section above, this algorithm is based on the "factorize.c" source code from the gmp library. Much of the code in RcppAlgos::primeFactorize is a straightforward translation from multiple precision C data types to standard C++ data types. A crucial part of the algorithm's efficiency is based on quickly determining primality, which is easily computed with gmp. However, with standard C++, this is quite challenging. Much of the research for RcppAlgos::primeFactorize was focused on developing an algorithm that could accurately and efficiently compute primality.
For more details, see the documentation for isPrimeRcpp.

Value

- Returns an unnamed vector if length(v) == 1 regardless of the value of namedList. If $v<$ $2^{31}$, the class of the returned vector will be integer, otherwise the class will be numeric.
- If length $(v)>1$, a named/unnamed list of vectors will be returned. If max (bound1, bound2) $<2^{31}$, the class of each vector will be integer, otherwise the class will be numeric.


## Note

The maximum value for each element in $v$ is $2^{53}-1$.

## Author(s)

Joseph Wood

## References

- Pollard's rho algorithm
- Miller-Rabin primality test
- Accurate Modular Arithmetic with Double Precision
- 53-bit significand precision


## See Also

primeFactorizeSieve, factorize, primeFactors

## Examples

```
## Get the prime factorization of a single number
primeFactorize(10^8)
## Or get the prime factorization of many numbers
set.seed(29)
myVec <- sample(-1000000:1000000, 1000)
system.time(pFacs <- primeFactorize(myVec))
## Return named list
pFacsWithNames <- primeFactorize(myVec, namedList = TRUE)
## Using nThreads
system.time(primeFactorize(myVec, nThreads = 2))
```

```
primeFactorizeSieve Generate Prime Factorization for Numbers in a Range
```


## Description

Generates the prime factorization of all numbers between bound1 and bound2 (if supplied) or all numbers up to bound1.

## Usage

primeFactorizeSieve(bound1, bound2 = NULL, namedList = FALSE, nThreads = NULL)

## Arguments

bound1 Positive integer or numeric value.
bound2 Positive integer or numeric value.
namedList Logical flag. If TRUE, a named list is returned. The default is FALSE.
nThreads $\quad$ Specific number of threads to be used. The default is NULL.

## Details

This function is useful when many prime factorizations are needed. Instead of generating the prime factorization on the fly, one can reference the indices/names of the generated list.

This algorithm benefits greatly from the fast integer division library 'libdivide'. The following is from http://libdivide.com/:

- "libdivide allows you to replace expensive integer divides with comparatively cheap multiplication and bitshifts. Compilers usually do this, but only when the divisor is known at compile time. libdivide allows you to take advantage of it at runtime. The result is that integer division can become faster - a lot faster."


## Value

Returns a named/unnamed list of integer vectors if $\max$ (bound1, bound2) $<2^{31}$, or a list of numeric vectors otherwise.

## Note

The maximum value for either of the bounds is $2^{53}-1$.

## Author(s)

Joseph Wood

## References

- Prime Factor
- ridiculousfish (author of libdivide)
- github.com/ridiculousfish/libdivide
- 53-bit significand precision


## See Also

## Examples

```
## Generate some random data
set.seed(28)
mySamp <- sample(10^5, 5*10^4)
## Generate prime factorizations up
## to 10^5 (max element from mySamp)
system.time(allPFacs <- primeFactorizeSieve(10^5))
## Use generated prime factorization for further
## analysis by accessing the index of allPFacs
for (s in mySamp) {
    pFac <- allPFacs[[s]]
    ## Continue algorithm
}
## Generating prime factorizations over
## a range is efficient as well
system.time(primeFactorizeSieve(10^12, 10^12 + 10^5))
## Set 'namedList' to TRUE to return a named list
primeFactorizeSieve(27, 30, namedList = TRUE)
## Using nThreads
system.time(primeFactorizeSieve(1e4, 5e4, nThreads = 2))
```

primeSieve Generate Prime Numbers

## Description

Implementation of the segmented sieve of Eratosthenes with wheel factorization. Generates all prime numbers between bound1 and bound2 (if supplied) or all primes up to bound1. See this stackoverflow post for an analysis on prime number generation efficiency in R: Generate a list of primes up to a certain number
The fundamental concepts of this algorithm are based off of the implementation by Kim Walisch found here: kimwalisch/primesieve.

## Usage

primeSieve(bound1, bound2 = NULL, nThreads = NULL)

## Arguments

bound1 Positive integer or numeric value.
bound2 Positive integer or numeric value.
nThreads Specific number of threads to be used. The default is NULL.

## Details

At the heart of this algorithm is the traditional sieve of Eratosthenes (i.e. given a prime $p$, mark all multiples of $p$ as composite), however instead of sieving the entire interval, we only consider small sub-intervals. The benefits of this method are two fold:

1. Reduction of the space complexity from $O(n)$, for the traditional sieve, to $O(\sqrt{n})$
2. Reduction of cache misses

The latter is of particular importance as cache memory is much more efficient and closer in proximity to the CPU than main memory. Reducing the size of the sieving interval allows for more effective utilization of the cache, which greatly impacts the overall efficiency.
Another optimization over the traditional sieve is the utilization of wheel factorization. With the traditional sieve of Eratosthenes, you typically check every odd index of your logical vector and if the value is true, you have found a prime. With wheel factorization using the first four primes (i.e. $2,3,5$, and 7 ) to construct your wheel (i.e. 210 wheel), you only have to check indices of your logical vector that are coprime to 210 (i.e. the product of the first four primes). As an example, with $n=10000$ and a 210 wheel, you only have to check 2285 indices vs. 5000 with the classical implementation.

## Value

Returns an integer vector if $\max$ (bound1, bound2) $<2^{31}$, or a numeric vector otherwise.

## Note

- It does not matter which bound is larger as the resulting primes will be between min(bound1, bound2) and max(bound1, bound2) if bound2 is provided.
- The maximum value for either of the bounds is $2^{53}-1$.


## Author(s)

Joseph Wood

## References

- primesieve (Fast C/C++ prime number generator)
- Sieve of Eratosthenes
- Wheel factorization
- 53-bit significand precision


## See Also

Primes

## Examples

```
## Primes up to a thousand
primeSieve(100)
## Primes between 42 and 17
primeSieve(42, 17)
## Equivalent to
primeSieve(17, 42)
## Primes up to one hundred million in no time
system.time(primeSieve(10^8))
## options(scipen = 50)
## Generate large primes over interval
system.time(myPs <- primeSieve(10^13+10^6, 10^13))
## Object created is small
object.size(myPs)
## Using nThreads
system.time(primeSieve(1e7, nThreads = 2))
```

```
stdThreadMax Max Number of Concurrent Threads
```


## Description

Wrapper of std::thread::hardware_concurrency(). As stated by cppreference, the returned value should be considered only a hint.

## Usage

stdThreadMax()

## Value

An integer representing the number of concurrent threads supported by the user implementation. If the value cannot be determined, 1 L is returned.

## See Also

detectCores

## Examples

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