Package ‘SMFilter’

December 12, 2018

Title Filtering Algorithms for the State Space Models on the Stiefel Manifold

Version 1.0.3

Description Provides the filtering algorithms for the state space models on the Stiefel manifold as well as the corresponding sampling algorithms for uniform, vector Langevin-Bingham and matrix Langevin-Bingham distributions on the Stiefel manifold.

Depends R (>= 3.0.0)

License GPL-3

Encoding UTF-8

LazyData true

RoxygenNote 6.1.1

URL https://github.com/yukai-yang/SMFilter

BugReports https://github.com/yukai-yang/SMFilter/issues

Suggests knitr, rmarkdown, ggplot2

VignetteBuilder knitr

NeedsCompilation no

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Repository CRAN

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R topics documented:

`FDist2` *Compute the squared Frobenius distance between two matrices.*

Description

This function Compute the squared Frobenius distance between two matrices.

Usage

```
FDist2(mX, mY)
```

Arguments

`mX`	a $p \times r$ matrix where $p \geq r$.
`mY`	another $p \times r$ matrix where $p \geq r$.

Details

The Frobenius distance between two matrices is defined to be

$$d(X, Y) = \sqrt{\mathrm{tr}\{A'A\}}$$

where $A = X - Y$.

The Frobenius distance is a possible measure of the distance between two points on the Stiefel manifold.

Value

the Frobenius distance.

Author(s)

Yukai Yang, `<yukai.yang@statistik.uu.se>`

Examples

```
FDist2(runif_sm(1,4,2)[1,,], runif_sm(1,4,2)[1,,])
```

FilterModel1 *Filtering algorithm for the type one model.*

Description

This function implements the filtering algorithm for the type one model. See Details part below.

Usage

```
FilterModel1(mY, mX, mZ, beta, mB = NULL, Omega, vD, U0,
  method = "max_1")
```

Arguments

mY	the matrix containing Y_t with dimension $T \times p$.
mX	the matrix containing X_t with dimension $T \times q_1$.
mZ	the matrix containing Z_t with dimension $T \times q_2$.
beta	the β matrix.
mB	the coefficient matrix $\boldsymbol{B}$ before mZ with dimension $p \times q_2$.
Omega	covariance matrix of the errors.
vD	vector of the diagonals of D.
U0	initial value of the alpha sequence.
method	a string representing the optimization method from c('max_1','max_2','max_3','min_1','min_2').

Details

The type one model on Stiefel manifold takes the form:

$$\begin{aligned} \boldsymbol{y}_t &= \boldsymbol{\alpha}_t \boldsymbol{\beta}' \boldsymbol{x}_t + \boldsymbol{B} \boldsymbol{z}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\alpha}_{t+1} | \boldsymbol{\alpha}_t &\sim ML(p, r, \boldsymbol{\alpha}_t \boldsymbol{D}) \end{aligned}$$

where $\boldsymbol{y}_t$ is a p-vector of the dependent variable, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ are explanatory variables wit dimension q_1 and q_2, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ have no overlap, matrix $\boldsymbol{B}$ is the coefficients for $\boldsymbol{z}_t$, $\boldsymbol{\varepsilon}_t$ is the error vector.

The matrices $\boldsymbol{\alpha}_t$ and $\boldsymbol{\beta}$ have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1. $\boldsymbol{\alpha}_t$ and $\boldsymbol{\beta}$ are both non-singular matrices. $\boldsymbol{\alpha}_t$ is time-varying while $\boldsymbol{\beta}$ is time-invariant.

Furthermore, $\boldsymbol{\alpha}_t$ fulfills the condition $\boldsymbol{\alpha}_t' \boldsymbol{\alpha}_t = \boldsymbol{I}_r$, and therefor it evolves on the Stiefel manifold.

$ML(p, r, \boldsymbol{\alpha}_t \boldsymbol{D})$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\alpha_{t+1}}) = \frac{\mathrm{etr}\left\{\boldsymbol{D} \boldsymbol{\alpha}_t' \boldsymbol{\alpha_{t+1}}\right\}}{{}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)}$$

where etr denotes $\exp(\mathrm{tr}())$, and ${}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)$ is the (0,1)-type hypergeometric function for matrix.

Value

an array aAlpha containing the modal orientations of alpha in the prediction step.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

```
iT = 50
ip = 2
ir = 1
iqx = 4
iqz=0
ik = 0
Omega = diag(ip)*.1

if(iqx==0) mX=NULL else mX = matrix(rnorm(iT*iqx),iT, iqx)
if(iqz==0) mZ=NULL else mZ = matrix(rnorm(iT*iqz),iT, iqz)
if(ik==0) mY=NULL else mY = matrix(0, ik, ip)

alpha_0 = matrix(c(runif_sm(num=1,ip=ip,ir=ir)), ip, ir)
beta = matrix(c(runif_sm(num=1,ip=ip*ik+iqx,ir=ir)), ip*ik+iqx, ir)
mB=NULL
vD = 100

ret = SimModel1(iT=iT, mX=mX, mZ=mZ, mY=mY, alpha_0=alpha_0, beta=beta, mB=mB, vD=vD, Omega=Omega)
mYY=as.matrix(ret$dData[,1:ip])
fil = FilterModel1(mY=mYY, mX=mX, mZ=mZ, beta=beta, mB=mB, Omega=Omega, vD=vD, U0=alpha_0)
```

FilterModel2 *Filtering algorithm for the type two model.*

Description

This function implements the filtering algorithm for the type two model. See Details part below.

Usage

```
FilterModel2(mY, mX, mZ, alpha, mB = NULL, Omega, vD, U0,
  method = "max_1")
```

Arguments

- `mY` the matrix containing Y_t with dimension $T \times p$.
- `mX` the matrix containing X_t with dimension $T \times q_1$.
- `mZ` the matrix containing Z_t with dimension $T \times q_2$.
- `alpha` the α matrix.
- `mB` the coefficient matrix $\boldsymbol{B}$ before `mZ` with dimension $p \times q_2$.
- `Omega` covariance matrix of the errors.
- `vD` vector of the diagonals of D.
- `U0` initial value of the alpha sequence.
- `method` a string representing the optimization method from c('max_1','max_2','max_3','min_1','min_2').

Details

The type two model on Stiefel manifold takes the form:

$$\begin{aligned} \boldsymbol{y}_t &= \boldsymbol{\alpha}\boldsymbol{\beta}_t'\boldsymbol{x}_t + \boldsymbol{B}'\boldsymbol{z}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\beta}_{t+1}|\boldsymbol{\beta}_t &\sim ML(q_1, r, \boldsymbol{\beta}_t\boldsymbol{D}) \end{aligned}$$

where $\boldsymbol{y}_t$ is a p-vector of the dependent variable, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ are explanatory variables wit dimension q_1 and q_2, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ have no overlap, matrix $\boldsymbol{B}$ is the coefficients for $\boldsymbol{z}_t$, $\boldsymbol{\varepsilon}_t$ is the error vector.

The matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}_t$ have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}_t$ are both non-singular matrices. $\boldsymbol{\beta}_t$ is time-varying while $\boldsymbol{\alpha}$ is time-invariant.

Furthermore, $\boldsymbol{\beta}_t$ fulfills the condition $\boldsymbol{\beta}_t'\boldsymbol{\beta}_t = \boldsymbol{I}_r$, and therefor it evolves on the Stiefel manifold.

$ML(p, r, \boldsymbol{\beta}_t\boldsymbol{D})$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\beta_{t+1}}) = \frac{\mathrm{etr}\left\{\boldsymbol{D}\boldsymbol{\beta}_t'\boldsymbol{\beta_{t+1}}\right\}}{{}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)}$$

where etr denotes $\exp(\mathrm{tr}())$, and ${}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)$ is the (0,1)-type hypergeometric function for matrix.

Value

an array `aAlpha` containing the modal orientations of alpha in the prediction step.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

```
iT = 50
ip = 2
ir = 1
iqx = 4
iqz=0
ik = 0
Omega = diag(ip)*.1

if(iqx==0) mX=NULL else mX = matrix(rnorm(iT*iqx),iT, iqx)
if(iqz==0) mZ=NULL else mZ = matrix(rnorm(iT*iqz),iT, iqz)
if(ik==0) mY=NULL else mY = matrix(0, ik, ip)

alpha = matrix(c(runif_sm(num=1,ip=ip,ir=ir)), ip, ir)
beta_0 = matrix(c(runif_sm(num=1,ip=ip*ik+iqx,ir=ir)), ip*ik+iqx, ir)
mB=NULL
vD = 100

ret = SimModel2(iT=iT, mX=mX, mZ=mZ, mY=mY, alpha=alpha, beta_0=beta_0, mB=mB, vD=vD)
mYY=as.matrix(ret$dData[,1:ip])
fil = FilterModel2(mY=mYY, mX=mX, mZ=mZ, alpha=alpha, mB=mB, Omega=Omega, vD=vD, U0=beta_0)
```

`rmLB_sm` *Sample from the matrix Langevin-Bingham on the Stiefel manifold.*

Description

This function draws a sample from the matrix Langevin-Bingham on the Stiefel manifold.

Usage

```
rmLB_sm(num, mJ, mH, mC, mX, ir)
```

Arguments

`num`	number of observations or sample size.
`mJ`	symmetric ip*ip matrix
`mH`	symmetric ir*ir matrix
`mC`	ip*ir matrix
`mX`	ip*ir matrix, the initial value
`ir`	ir

Details

The matrix Langevin-Bingham distribution on the Stiefel manifold has the density kernel:

$$f(X) \propto \mathrm{etr}\{HX'JX + C'X\}$$

where X satisfies $X'X = I_r$, and H and J are symmetric matrices.

Value

an array containing a sample of draws from the matrix Langevin-Bingham on the Stiefel manifold.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

#' @section References: Hoff, P. D. (2009) "Simulation of the Matrix Bingham—von Mises—Fisher Distribution, With Applications to Multivariate and Relational Data", Journal of Computational and Graphical Statistics, Vol. 18, pp. 438-456.

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runif_sm *Sample from the uniform distribution on the Stiefel manifold.*

Description

This function draws a sample from the uniform distribution on the Stiefel manifold.

Usage

```
runif_sm(num, ip, ir)
```

Arguments

num	number of observations or sample size.
ip	the first dimension p of the matrix.
ir	the second dimension r of the matrix.

Details

The Stiefel manifold with dimension p and r ($p \geq r$) is a space whose points are r-frames in R^p. A set of r orthonormal vectors in R^p is called an r-frame in R^p. The Stiefel manifold is a collection of $p \times r$ full rank matrices X such that $X'X = I_r$.

Value

an array with dimension `num`, `ip` and `ir` containing a sample of draws from the uniform distribution on the Stiefel manifold.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

```
runif_sm(10,4,2)
```

---

rvlb_sm *Sample from the vector Langevin-Bingham on the Stiefel manifold.*

Description

This function draws a sample from the vector Langevin-Bingham on the Stiefel manifold.

Usage

```
rvlb_sm(num, mA, vc, vx)
```

Arguments

num	number of observations or sample size.
mA	the matrix A which is symmetric ip*ip matrix.
vc	the vector c with dimension ip.
vx	the vector x, the initial value.

Details

The vector Langevin-Bingham distribution on the Stiefel manifold has the density kernel:

$$f(X) \propto \mathrm{etr}\{x'Ax + c'x\}$$

where x satisfies $x'x = 1$, and A is a symmetric matrix.

Value

an array containing a sample of draws from the vector Langevin-Bingham on the Stiefel manifold.

References

Hoff, P. D. (2009) "Simulation of the Matrix Bingham—von Mises—Fisher Distribution, With Applications to Multivariate and Relational Data", Journal of Computational and Graphical Statistics, Vol. 18, pp. 438-456.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

SimModel1 *Simulate from the type one state-space Model on Stiefel manifold.*

Description

This function simulates from the type one model on Stiefel manifold. See Details part below.

Usage

```
SimModel1(iT, mX = NULL, mZ = NULL, mY = NULL, alpha_0, beta,
  mB = NULL, Omega = NULL, vD, burnin = 100)
```

Arguments

`iT`	the sample size.
`mX`	the matrix containing X_t with dimension $T \times q_1$.
`mZ`	the matrix containing Z_t with dimension $T \times q_2$.
`mY`	initial values of the dependent variable for `ik-1` up to 0. If `mY = NULL`, then no lagged dependent variables in regressors.
`alpha_0`	the initial alpha, $p \times r$.
`beta`	the β matrix, iqx+ip*ik, y_1,t-1,y_1,t-2,...,y_2,t-1,y_2,t-2,...
`mB`	the coefficient matrix $\boldsymbol{B}$ before `mZ` with dimension $p \times q_2$.
`Omega`	covariance matrix of the errors.
`vD`	vector of the diagonals of D.
`burnin`	burn-in sample size (matrix Langevin).

Details

The type one model on Stiefel manifold takes the form:

$$\begin{aligned} \boldsymbol{y}_t &= \boldsymbol{\alpha}_t \boldsymbol{\beta}' \boldsymbol{x}_t + \boldsymbol{B}\boldsymbol{z}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\alpha}_{t+1}|\boldsymbol{\alpha}_t &\sim ML(p, r, \boldsymbol{\alpha}_t \boldsymbol{D}) \end{aligned}$$

where $\boldsymbol{y}_t$ is a p-vector of the dependent variable, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ are explanatory variables wit dimension q_1 and q_2, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ have no overlap, matrix $\boldsymbol{B}$ is the coefficients for $\boldsymbol{z}_t$, $\boldsymbol{\varepsilon}_t$ is the error vector.

The matrices $\boldsymbol{\alpha}_t$ and $\boldsymbol{\beta}$ have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1. $\boldsymbol{\alpha}_t$ and $\boldsymbol{\beta}$ are both non-singular matrices. $\boldsymbol{\alpha}_t$ is time-varying while $\boldsymbol{\beta}$ is time-invariant.

Furthermore, $\boldsymbol{\alpha}_t$ fulfills the condition $\boldsymbol{\alpha}_t' \boldsymbol{\alpha}_t = \boldsymbol{I}_r$, and therefor it evolves on the Stiefel manifold.

$ML(p, r, \boldsymbol{\alpha}_t \boldsymbol{D})$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\alpha_{t+1}}) = \frac{\mathrm{etr}\left\{\boldsymbol{D}\boldsymbol{\alpha}_t' \boldsymbol{\alpha_{t+1}}\right\}}{{}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)}$$

where etr denotes $\exp(\mathrm{tr}())$, and ${}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)$ is the (0,1)-type hypergeometric function for matrix.

Note that the function does not add intercept automatically.

Value

A list containing the sampled data and the dynamics of alpha.

The object is a list containing the following components:

`dData` a data.frame of the sampled data

`aAlpha` an array of the $\boldsymbol{\alpha}_t$ with the dimension $T \times p \times r$

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

```
iT = 50 # sample size
ip = 2 # dimension of the dependent variable
ir = 1 # rank number
iqx=2 # number of variables in X
iqz=2 # number of variables in Z
ik = 1 # lag length

if(iqx==0) mX=NULL else mX = matrix(rnorm(iT*iqx),iT, iqx)
if(iqz==0) mZ=NULL else mZ = matrix(rnorm(iT*iqz),iT, iqz)
if(ik==0) mY=NULL else mY = matrix(0, ik, ip)

alpha_0 = matrix(c(runif_sm(num=1,ip=ip,ir=ir)), ip, ir)
beta = matrix(c(runif_sm(num=1,ip=ip*ik+iqx,ir=ir)), ip*ik+iqx, ir)
if(ip*ik+iqz==0) mB=NULL else mB = matrix(c(runif_sm(num=1,ip=(ip*ik+iqz)*ip,ir=1)), ip, ip*ik+iqz)
vD = 50

ret = SimModel1(iT=iT, mX=mX, mZ=mZ, mY=mY, alpha_0=alpha_0, beta=beta, mB=mB, vD=vD)
```

`SimModel2` *Simulate from the type two state-space Model on Stiefel manifold.*

Description

This function simulates from the type two model on Stiefel manifold. See Details part below.

Usage

```
SimModel2(iT, mX = NULL, mZ = NULL, mY = NULL, beta_0, alpha,
  mB = NULL, Omega = NULL, vD, burnin = 100)
```

Arguments

`iT`	the sample size.
`mX`	the matrix containing X_t with dimension $T \times q_1$.
`mZ`	the matrix containing Z_t with dimension $T \times q_2$.
`mY`	initial values of the dependent variable for `ik-1` up to 0. If `mY = NULL`, then no lagged dependent variables in regressors.
`beta_0`	the initial beta, iqx+ip*ik, y_1,t-1,y_1,t-2,...,y_2,t-1,y_2,t-2,....
`alpha`	the α matrix, $p \times r$.
`mB`	the coefficient matrix $\boldsymbol{B}$ before `mZ` with dimension $p \times q_2$.
`Omega`	covariance matrix of the errors.
`vD`	vector of the diagonals of D.
`burnin`	burn-in sample size (matrix Langevin).

Details

The type two model on Stiefel manifold takes the form:

$$\begin{aligned}\boldsymbol{y}_t &= \boldsymbol{\alpha}\boldsymbol{\beta}_t'\boldsymbol{x}_t + \boldsymbol{B}'\boldsymbol{z}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\beta}_{t+1}|\boldsymbol{\beta}_t &\sim ML(q_1, r, \boldsymbol{\beta}_t\boldsymbol{D})\end{aligned}$$

where $\boldsymbol{y}_t$ is a p-vector of the dependent variable, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ are explanatory variables wit dimension q_1 and q_2, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ have no overlap, matrix $\boldsymbol{B}$ is the coefficients for $\boldsymbol{z}_t$, $\boldsymbol{\varepsilon}_t$ is the error vector.

The matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}_t$ have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}_t$ are both non-singular matrices. $\boldsymbol{\beta}_t$ is time-varying while $\boldsymbol{\alpha}$ is time-invariant.

Furthermore, $\boldsymbol{\beta}_t$ fulfills the condition $\boldsymbol{\beta}_t'\boldsymbol{\beta}_t = \boldsymbol{I}_r$, and therefor it evolves on the Stiefel manifold.

$ML(p, r, \boldsymbol{\beta}_t\boldsymbol{D})$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\beta_{t+1}}) = \frac{\mathrm{etr}\left\{\boldsymbol{D}\boldsymbol{\beta}_t'\boldsymbol{\beta_{t+1}}\right\}}{{}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)}$$

where etr denotes $\exp(\mathrm{tr}())$, and ${}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)$ is the (0,1)-type hypergeometric function for matrix.

Note that the function does not add intercept automatically.

Value

A list containing the sampled data and the dynamics of beta.

The object is a list containing the following components:

`dData`	a data.frame of the sampled data
`aBeta`	an array of the $\boldsymbol{\beta}_t$ with the dimension $T \times q_1 \times r$

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

```
iT = 50
ip = 2
ir = 1
iqx =3
iqz=2
ik = 1

if(iqx==0) mX=NULL else mX = matrix(rnorm(iT*iqx),iT, iqx)
if(iqz==0) mZ=NULL else mZ = matrix(rnorm(iT*iqz),iT, iqz)
if(ik==0) mY=NULL else mY = matrix(0, ik, ip)

alpha = matrix(c(runif_sm(num=1,ip=ip,ir=ir)), ip, ir)
beta_0 = matrix(c(runif_sm(num=1,ip=ip*ik+iqx,ir=ir)), ip*ik+iqx, ir)
if(ip*ik+iqz==0) mB=NULL else mB = matrix(c(runif_sm(num=1,ip=(ip*ik+iqz)*ip,ir=1)), ip, ip*ik+iqz)
vD = 50

ret = SimModel2(iT=iT, mX=mX, mZ=mZ, mY=mY, alpha=alpha, beta_0=beta_0, mB=mB, vD=vD)
```

`SMFilter` *SMFilter: a package implementing the filtering algorithms for the state-space models on the Stiefel manifold.*

Description

The package implements the filtering algorithms for the state-space models on the Stiefel manifold. It also implements sampling algorithms for uniform, vector Langevin-Bingham and matrix Langevin-Bingham distributions on the Stiefel manifold.

Details

Two types of the state-space models on the Stiefel manifold are considered.

The type one model on Stiefel manifold takes the form:

$$\begin{aligned} \boldsymbol{y}_t &= \boldsymbol{\alpha}_t \boldsymbol{\beta}' \boldsymbol{x}_t + \boldsymbol{B} \boldsymbol{z}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\alpha}_{t+1} | \boldsymbol{\alpha}_t &\sim ML(p, r, \boldsymbol{\alpha}_t \boldsymbol{D}) \end{aligned}$$

where $\boldsymbol{y}_t$ is a p-vector of the dependent variable, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ are explanatory variables wit dimension q_1 and q_2, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ have no overlap, matrix $\boldsymbol{B}$ is the coefficients for $\boldsymbol{z}_t$, $\boldsymbol{\varepsilon}_t$ is the error vector.

The matrices $\boldsymbol{\alpha}_t$ and $\boldsymbol{\beta}$ have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1. $\boldsymbol{\alpha}_t$ and $\boldsymbol{\beta}$ are both non-singular matrices. $\boldsymbol{\alpha}_t$ is time-varying while $\boldsymbol{\beta}$ is time-invariant.

Furthermore, $\boldsymbol{\alpha}_t$ fulfills the condition $\boldsymbol{\alpha}_t'\boldsymbol{\alpha}_t = \boldsymbol{I}_r$, and therefor it evolves on the Stiefel manifold.

$ML(p, r, \boldsymbol{\alpha}_t \boldsymbol{D})$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\alpha_{t+1}}) = \frac{\mathrm{etr}\left\{\boldsymbol{D}\boldsymbol{\alpha}_t'\boldsymbol{\alpha_{t+1}}\right\}}{{}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)}$$

where etr denotes $\exp(\mathrm{tr}())$, and ${}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)$ is the (0,1)-type hypergeometric function for matrix.

The type two model on Stiefel manifold takes the form:

$$\begin{aligned} \boldsymbol{y}_t &= \boldsymbol{\alpha}\boldsymbol{\beta}_t'\boldsymbol{x}_t + \boldsymbol{B}'\boldsymbol{z}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\beta}_{t+1}|\boldsymbol{\beta}_t &\sim ML(q_1, r, \boldsymbol{\beta}_t \boldsymbol{D}) \end{aligned}$$

where $\boldsymbol{y}_t$ is a p-vector of the dependent variable, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ are explanatory variables wit dimension q_1 and q_2, $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ have no overlap, matrix $\boldsymbol{B}$ is the coefficients for $\boldsymbol{z}_t$, $\boldsymbol{\varepsilon}_t$ is the error vector.

The matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}_t$ have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}_t$ are both non-singular matrices. $\boldsymbol{\beta}_t$ is time-varying while $\boldsymbol{\alpha}$ is time-invariant.

Furthermore, $\boldsymbol{\beta}_t$ fulfills the condition $\boldsymbol{\beta}_t'\boldsymbol{\beta}_t = \boldsymbol{I}_r$, and therefor it evolves on the Stiefel manifold.

$ML(p, r, \boldsymbol{\beta}_t \boldsymbol{D})$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\beta_{t+1}}) = \frac{\mathrm{etr}\left\{\boldsymbol{D}\boldsymbol{\beta}_t'\boldsymbol{\beta_{t+1}}\right\}}{{}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)}$$

where etr denotes $\exp(\mathrm{tr}())$, and ${}_0F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)$ is the (0,1)-type hypergeometric function for matrix.

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References

Yang, Yukai and Bauwens, Luc. (2018) "State-Space Models on the Stiefel Manifold with a New Approach to Nonlinear Filtering", Econometrics, 6(4), 48.

Simulation

`SimModel1` simulate from the type one state-space model on the Stiefel manifold.

`SimModel2` simulate from the type two state-space model on the Stiefel manifold.

Filtering

`FilterModel1` filtering algorithm for the type one model.

`FilterModel2` filtering algorithm for the type two model.

Sampling

runif_sm sample from the uniform distribution on the Stiefel manifold.

rvlb_sm sample from the vector Langevin-Bingham distribution on the Stiefel manifold.

rmLB_sm sample from the matrix Langevin-Bingham distribution on the Stiefel manifold.

Other Functions

version shows the version number and some information of the package.

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version *Show the version number of some information.*

Description

This function shows the version number and some information of the package.

Usage

```
version()
```

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

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