

On the usage of the `pbkrtest` package

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Contents

1	Introduction	1
2	Kenward–Roger approach	3
3	Parametric bootstrap	4
A	Matrices for random effects	5

1 Introduction

The `shoes` data is a list of two vectors, giving the wear of shoes of materials A and B for one foot each of ten boys.

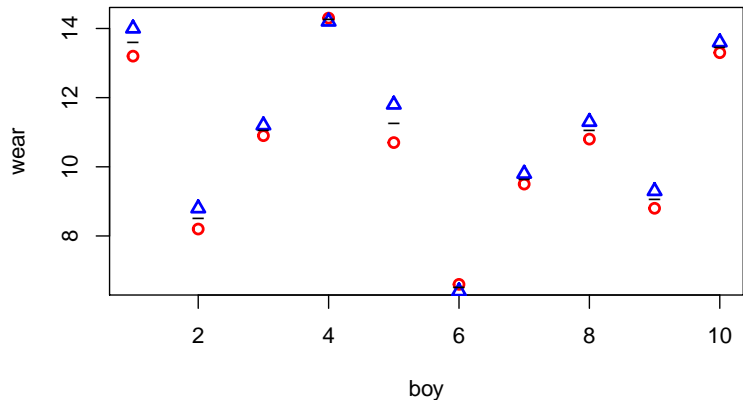
```
R> data(shoes, package="MASS")
R> shoes
```

```
$A
 [1] 13.2  8.2 10.9 14.3 10.7  6.6  9.5 10.8  8.8 13.3
```

```
$B
 [1] 14.0  8.8 11.2 14.2 11.8  6.4  9.8 11.3  9.3 13.6
```

A plot clearly reveals that boys wear their shoes differently.

```
R> plot(A~1, data=shoes, col="red", lwd=2, pch=1, ylab="wear", xlab="boy")
R> points(B~1, data=shoes, col="blue", lwd=2, pch=2)
R> points(I((A+B)/2)~1, data=shoes, pch="-", lwd=2)
```



One option for testing the effect of materials is to make a paired t -test. The following forms are equivalent:

```
R> r1<-t.test(shoes$A, shoes$B, paired=T)
R> r2<-t.test(shoes$A-shoes$B)
R> r1
```

Paired t-test

```
data: shoes$A and shoes$B
t = -3.3489, df = 9, p-value = 0.008539
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.6869539 -0.1330461
sample estimates:
mean of the differences
 -0.41
```

To work with data in a mixed model setting we create a dataframe, and for later use we also create an imbalanced version of data:

```
R> boy <- rep(1:10,2)
R> boyf<- factor(letters[boy])
R> mat <- factor(c(rep("A", 10), rep("B",10)))
R> ## Balanced data:
R> shoe.b <- data.frame(wear=unlist(shoes), boy=boy, boyf=boyf, mat=mat)
R> head(shoe.b)
```

	wear	boy	boyf	mat
A1	13.2	1	a	A
A2	8.2	2	b	A
A3	10.9	3	c	A
A4	14.3	4	d	A
A5	10.7	5	e	A
A6	6.6	6	f	A

```
R> ## Imbalanced data; delete (boy=1, mat=1) and (boy=2, mat=b)
R> shoe.i <- shoe.b[-c(1,12),]
```

We fit models to the two datasets:

```
R> lmm1.b <- lmer( wear ~ mat + (1|boyf), data=shoe.b )
R> lmm0.b <- update( lmm1.b, .~. - mat)
R> lmm1.i <- lmer( wear ~ mat + (1|boyf), data=shoe.i )
R> lmm0.i <- update(lmm1.i, .~. - mat)
```

The asymptotic likelihood ratio test shows stronger significance than the t -test:

```
R> anova( lmm1.b, lmm0.b, test="Chisq" ) ## Balanced data
```

Data: shoe.b

Models:

```
lmm0.b: wear ~ (1 | boyf)
```

```
lmm1.b: wear ~ mat + (1 | boyf)
```

	npar	AIC	BIC	logLik	deviance	Chisq	Df	Pr(>Chisq)
lmm0.b	3	67.909	70.896	-30.955	61.909			
lmm1.b	4	61.817	65.800	-26.909	53.817	8.092	1	0.004446 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
R> anova( lmm1.i, lmm0.i, test="Chisq" ) ## Imbalanced data
```

Data: shoe.i

Models:

```
lmm0.i: wear ~ (1 | boyf)
```

```
lmm1.i: wear ~ mat + (1 | boyf)
```

	npar	AIC	BIC	logLik	deviance	Chisq	Df	Pr(>Chisq)
lmm0.i	3	63.869	66.540	-28.934	57.869			
lmm1.i	4	60.777	64.339	-26.389	52.777	5.092	1	0.02404 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

2 Kenward–Roger approach

The Kenward–Roger approximation is exact for the balanced data in the sense that it produces the same result as the paired t -test.

```
R> ( kr.b<-KRmodcomp(lmm1.b, lmm0.b) )
```

```
large : wear ~ mat + (1 | boyf)
```

```
small : wear ~ (1 | boyf)
```

	stat	ndf	ddf	F.scaling	p.value
Ftest	11.215	1.000	9.000	1	0.008539 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
R> summary( kr.b )
```

F-test with Kenward–Roger approximation; time: 0.18 sec

```
large : wear ~ mat + (1 | boyf)
```

```
small : wear ~ (1 | boyf)
```

	stat	ndf	ddf	F.scaling	p.value
Ftest	11.215	1.000	9.000	1	0.008539 **
FtestU	11.215	1.000	9.000		0.008539 **

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Relevant information can be retrieved with

```
R> getKR(kr.b, "ddf")
```

```
[1] 9
```

For the imbalanced data we get

```
R> ( kr.i<-KRmodcomp(lmm1.i, lmm0.i) )
```

```
large : wear ~ mat + (1 | boyf)
```

```
small : wear ~ (1 | boyf)
```

```
          stat      ndf      ddf F.scaling p.value
Ftest 5.9893 1.0000 7.0219          1 0.04418 *
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Notice that this result is similar to but not identical to the paired t -test when the two relevant boys are removed:

```
R> shoes2 <- list(A=shoes$A[-(1:2)], B=shoes$B[-(1:2)])
```

```
R> t.test(shoes2$A, shoes2$B, paired=T)
```

```
Paired t-test
```

```
data: shoes2$A and shoes2$B
```

```
t = -2.3878, df = 7, p-value = 0.04832
```

```
alternative hypothesis: true difference in means is not equal to 0
```

```
95 percent confidence interval:
```

```
-0.671721705 -0.003278295
```

```
sample estimates:
```

```
mean of the differences
```

```
-0.3375
```

3 Parametric bootstrap

Parametric bootstrap provides an alternative but many simulations are often needed to provide credible results (also many more than shown here; in this connection it can be useful to exploit that computings can be made en parallel, see the documentation):

```
R> ( pb.b <- PBmodcomp(lmm1.b, lmm0.b, nsim=500, cl=2) )
```

```
Bootstrap test; time: 4.42 sec; samples: 500; extremes: 1;
```

```
large : wear ~ mat + (1 | boyf)
```

```
wear ~ (1 | boyf)
```

```
          stat df  p.value
LRT      8.1197  1 0.004379 **
PBtest  8.1197   0.003992 **
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
R> summary( pb.b )
```

```

Bootstrap test; time: 4.42 sec; samples: 500; extremes: 1;
large : wear ~ mat + (1 | boyf)
wear ~ (1 | boyf)
      stat    df    ddf p.value
LRT    8.1197 1.0000      0.004379 **
PBtest  8.1197                0.003992 **
Gamma   8.1197                0.004411 **
Bartlett 7.9409 1.0000      0.004833 **
F       8.1197 1.0000 90.806 0.005417 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

For the imbalanced data, the result is similar to the result from the paired t
test.

```

```

R> ( pb.i<-PBmodcomp(lmm1.i, lmm0.i, nsim=500, cl=2) )

Bootstrap test; time: 4.42 sec; samples: 500; extremes: 22;
large : wear ~ mat + (1 | boyf)
wear ~ (1 | boyf)
      stat df p.value
LRT    5.1151 1 0.02372 *
PBtest 5.1151  0.04591 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

R> summary( pb.i )

Bootstrap test; time: 4.42 sec; samples: 500; extremes: 22;
large : wear ~ mat + (1 | boyf)
wear ~ (1 | boyf)
      stat    df    ddf p.value
LRT    5.1151 1.0000      0.02372 *
PBtest  5.1151                0.04591 *
Gamma   5.1151                0.05198 .
Bartlett 4.2362 1.0000      0.03957 *
F       5.1151 1.0000 11.64 0.04372 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

A Matrices for random effects

The matrices involved in the random effects can be obtained with

```

R> shoe3 <- subset(shoe.b, boy<=5)
R> shoe3 <- shoe3[order(shoe3$boy), ]
R> lmm1 <- lmer( wear ~ mat + (1|boyf), data=shoe3 )
R> str( SG <- get_SigmaG( lmm1 ), max=2)

List of 3
 $ Sigma :Formal class 'dgCMatrix' [package "Matrix"] with 6 slots
 $ G     :List of 2
 ..$ :Formal class 'dgCMatrix' [package "Matrix"] with 6 slots

```

```

..$ :Formal class 'dgCMatrix' [package "Matrix"] with 6 slots
$ n.ggamma: int 2

R> round( SG$Sigma*10 )

10 x 10 sparse Matrix of class "dgCMatrix"

A1 53 52 . . . . .
B1 52 53 . . . . .
A2 . . 53 52 . . . . .
B2 . . 52 53 . . . . .
A3 . . . . 53 52 . . . . .
B3 . . . . 52 53 . . . . .
A4 . . . . . 53 52 . . . . .
B4 . . . . . 52 53 . . . . .
A5 . . . . . . 53 52 . . . . .
B5 . . . . . . 52 53 . . . . .

R> SG$G

[[1]]
10 x 10 sparse Matrix of class "dgCMatrix"

A1 1 1 . . . . .
B1 1 1 . . . . .
A2 . . 1 1 . . . . .
B2 . . 1 1 . . . . .
A3 . . . . 1 1 . . . . .
B3 . . . . 1 1 . . . . .
A4 . . . . . 1 1 . . . . .
B4 . . . . . 1 1 . . . . .
A5 . . . . . . 1 1 . . . . .
B5 . . . . . . 1 1 . . . . .

[[2]]
10 x 10 sparse Matrix of class "dgCMatrix"

[1,] 1 . . . . .
[2,] . 1 . . . . .
[3,] . . 1 . . . . .
[4,] . . . 1 . . . . .
[5,] . . . . 1 . . . . .
[6,] . . . . . 1 . . . . .
[7,] . . . . . . 1 . . . . .
[8,] . . . . . . . 1 . . . . .
[9,] . . . . . . . . 1 . . . . .
[10,] . . . . . . . . . 1 . . . . .

```