

# Package ‘scaleAlign’

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**Type** Package

**Title** Scale Alignment for Between-Items Multidimensional Rasch Family Models

**Version** 1.0.0.0

**Description** Scale alignment is a new procedure for rescaling dimensions of between-items multidimensional Rasch family models so that dimensions scores can be compared directly (Feuerstahler & Wilson, 2019; under review) <doi:10.1111/jedm.12209>. This package includes functions for implementing delta-dimensional alignment (DDA) and logistic regression alignment (LRA) for dichotomous or polytomous data. This function also includes a wrapper for models fit using the 'TAM' package.

**License** GPL-3

**Suggests** TAM, MASS, catR

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**LazyData** true

**RoxygenNote** 7.1.1

**NeedsCompilation** no

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align

Scale Alignment Wrapper Function for 'TAM' Objects

**Description**

Apply scale alignment methods to models previously fit with tam.mml in the 'TAM' package.

**Usage**

```
align(mod, method = "best", refdim = 1)
```

**Arguments**

mod	Fitted model of class tam.mml. Importantly, mod\$irtmodel must be either "1PL", "PCM", or "PCM2"
method	Either "DDA1", "DDA2", "LRA", or "best", see details
refdim	Which is the reference dimension (unchanged during alignment)

**Details**

Scales can be said to be aligned if the item sufficient statistics imply the same item parameter estimates, regardless of dimension. Scale alignment is currently defined only for Rasch family models with between-items multidimensionality (i.e., each scored item belongs to exactly one dimension).

**MODEL PARAMETERIZATIONS**

The partial credit model is a general Rasch family model for polytomous item responses. Within 'TAM', the partial credit model can be parameterized in two ways. If a 'TAM' model is fit with the option irtmodel = "PCM", then the following model is specified for an item with  $m + 1$  response categories:

$$\log\left(\frac{P(x|\theta_d)}{P(x-1|\theta_d)}\right) = \alpha_d\theta_d - \xi_{i(d)x}$$

for response category  $x = 1, \dots, m$ , and

$$P(x=0|\theta_d) = \frac{1}{\sum_{j=0}^m \exp \sum_{k=0}^j (\alpha_d\theta_d - \xi_{i(d)k})}$$

for response category  $x = 0$ .  $\alpha_d$  is a dimension steepness parameter, typically fixed to 1,  $\theta_d$  is a latent variable on dimension  $d$ , and  $\xi_{i(d)x}$  is a step parameter for item step  $x$  on item  $i$  belonging to dimension  $d$ .

If instead a TAM model is fit with the option irtmodel = "PCM2", the model is specified as

$$\log\left(\frac{P(x|\theta_d)}{P(x-1|\theta_d)}\right) = \alpha_d\theta_d - \delta_{i(d)} + \tau_{i(d)x}.$$

**MODEL TRANSFORMATIONS**

Under Rasch family models, the latent trait metric can be linearly transformed. For each dimension  $d$  the parameters on the transformed metric (denoted by the  $\sim$  symbol) are found through the transformation parameters  $r_d$  and  $s_d$  as described by the following equations:

$$\tilde{\theta}_d = r_d \theta_d + s_d$$

$$\tilde{\alpha}_d = \alpha_d / r_d$$

$$\tilde{\xi}_{i(d)x} = \xi_{i(d)x} + \alpha_d s_d / r_d$$

$$\tilde{\delta}_{i(d)} = \delta_{i(d)} + \alpha_d s_d / r_d$$

$$\tilde{\tau}_{i(d)x} = \tau_{i(d)x}$$

#### SUFFICIENT STATISTICS

Under Rasch family models, the item sufficient statistics are the number of examinees that score in response category  $x$  or higher,  $x = 1, \dots, m$ . For the purpose of scale alignment, we consider sufficient statistics to be the proportion of examinees that score in response category  $x$  or higher. This definition allows for scale alignment in the presence of missing data.

#### THURSTONE THRESHOLDS

Scales are aligned if the same sufficient statistics imply the same item parameters, regardless of dimension. The success of scale alignment is difficult to assess because the item sufficient statistics typically differ across items and dimensions. Under the Rasch model for binary item responses, the success of scale alignment can be assessed by looking at the rank-order correlation (e.g., Kendall's tau) between item sufficient statistics and item parameter estimates.

However, under the partial credit model, item sufficient statistics need not be monotonically related to estimated item parameters. Under this model, we can assess the quality of scale alignment by taking the rank-order correlation between item sufficient statistics and Thurstone thresholds. Thurstone thresholds are defined as the  $\theta$  value at which the probability of responding in category  $x$  or higher equals .5. Thurstone thresholds, in most cases, will be monotonically related to item sufficient statistics (within dimensions). Note that the item difficulty estimates under the Rasch model for binary items are also Thurstone thresholds.

#### ALIGNMENT METHODS

Two types of scale alignment methods have been developed.

The first class of methods, historically called delta-dimensional alignment (DDA), requires fitting both a multidimensional model and a model in which all items belong to a single dimension. With these two sets of parameter estimates, the transformation parameters  $r_d$  and  $s_d$  are then found so that, for each dimension, the means and standard deviation of parameters from the transformed multidimensional models equal the means and standard deviations of parameters from the unidimensional model. Under the ordinary Rasch model, the estimated item difficulties can be used for transformation (which is done if either method "DDA1" or "DDA2" is selected). Under the partial credit model, either the  $\delta$  parameters or the Thurstone thresholds from the two models may be

used within the DDA (note that DDA using item  $\xi$  parameters tends to be unsuccessful). Method "DDA1" uses the item  $\delta$  parameters, and method "DDA2" uses the Thurstone thresholds. If all items are binary, "DDA1" and "DDA2" are identical.

The second class of methods, called logistic regression alignment (LRA), requires fitting a logistic regression between item sufficient statistics and Thurstone thresholds for each dimension. The fitted logistic regression coefficients can then be used to estimate  $r_d$  and  $s_d$  so that the same logistic regression curve expresses the relationship between sufficient statistics and Thurstone thresholds for all dimensions.

For either the DDA or LRA method, a reference dimension (by default, the first dimension) is specified such that  $r_d = 1$  and  $s_d = 0$  for the reference dimension.

### Value

Aligned tam.mml object with the following added list items:

method	Alignment method: "DDA1", "DDA2", or "LRA"
rhat	Vector of estimated scaling parameters $r$ , see details
shat	Vector of estimated shift parameters $s$ , see details
cor_before	Kendall's rank-order correlation between sufficient statistics and Thurstone thresholds before alignment
cor_after	Kendall's rank-order correlation between sufficient statistics and Thurstone thresholds after alignment

### References

- Feuerstahler, L. M., & Wilson, M. (2019). Scale alignment in between-item multidimensional Rasch models. *Journal of Educational Measurement*, 56(2), 280–301. <doi: 10.1111/jedm.12209>
- Feuerstahler, L. M., & Wilson, M. (under review). Scale alignment in the between-items multidimensional partial credit model.

### Examples

```
## Example 1: binary item response data

## generate data for a 2-dimensional model with 10 items on each dimension

if(require(TAM)){

set.seed(2524)

diff_1 <- rnorm(10)
diff_2 <- rnorm(10)

N <- 500

th <- MASS::mvrnorm(N, mu = c(0, -1),
                    Sigma = matrix(c(1, .5 * 2, .5 * 2, 4), nrow = 2))
```

```

probs_1 <- 1 / (1 + exp(-outer(th[, 1], diff_1, "-")))
probs_2 <- 1 / (1 + exp(-outer(th[, 2], diff_2, "-")))

probs <- cbind(probs_1, probs_2)

dat <- apply(probs, 2, function(p) as.numeric(p > runif(N)))

Q <- cbind(c(rep(1, 10), rep(0, 10)),
           c(rep(0, 10), rep(1, 10)))

# fit the model

mod <- TAM::tam.mml(resp = dat, irtmodel = "1PL", Q = Q)

# align the model

mod_aligned <- align(mod)

## check alignment success

mod_aligned$cor_before
mod_aligned$cor_after

## view "best" alignment method
mod_aligned$method

## view alignment parameters

mod_aligned$rhat
mod_aligned$shat

}

## Example 2: Partial Credit Model

# generate 3-category data for a 2-dimensional model with 5 items on each dimension

set.seed(8491)

N <- 500

th <- MASS::mvrnorm(N, mu = c(0, 0),
                   Sigma = matrix(c(1, .5 * 2, .5 * 2, 4), nrow = 2))

xi_1 <- rnorm(5)
xi_1 <- cbind(xi_1, xi_1 + rnorm(5, mean = 1, sd = .5))

xi_2 <- rnorm(5)
xi_2 <- cbind(xi_2, xi_2 + rnorm(5, mean = 1, sd = .5))

dat1 <- catR::genPattern(th[, 1], it = xi_1, model = "PCM")

```

```

dat2 <- catR::genPattern(th[, 2], it = xi_2, model = "PCM")

dat <- cbind(dat1, dat2)

Q <- cbind(c(rep(1, 5), rep(0, 5)),
           c(rep(0, 5), rep(1, 5)))

## fit the model using both parameterizations

mod1 <- TAM::tam.mml(resp = dat, irtmodel = "PCM", Q = Q)
mod2 <- TAM::tam.mml(resp = dat, irtmodel = "PCM2", Q = Q)

## align the models
mod1_aligned <- align(mod1)
mod2_aligned <- align(mod2)

## check alignment success

mod1_aligned$cor_before
mod1_aligned$cor_after

mod2_aligned$cor_before
mod2_aligned$cor_after

## view "best" alignment method
mod1_aligned$method
mod2_aligned$method

## view alignment parameters

mod1_aligned$rhat
mod1_aligned$shat
mod2_aligned$rhat
mod2_aligned$shat

```

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align\_raw

*Scale Alignment Methods*


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### Description

Apply scale alignment methods to sets of item parameters and other model information provided by the user. These functions may be used to apply scale alignment to output from other item parameter estimation programs such as ConQuest. Note that `dda1` and `dda2` both require user-provided unidimensional and multidimensional item parameters. The `lra` function requires user-provided sufficient statistics, thresholds, and the number of observations and response categories per item. The function `lra2` is a wrapper for `lra` that computes some needed quantities from a user-provided data set.

**Usage**

```
dda1(
  multi_pars,
  uni_pars,
  itemtype,
  item_ind,
  dim_ind_i,
  refdim = 1,
  alpha = 1
)
```

```
dda2(
  multi_pars,
  uni_pars,
  itemtype,
  item_ind,
  dim_ind_i,
  refdim = 1,
  alpha = 1
)
```

```
lra(
  multi_pars,
  itemtype,
  ss,
  nobs,
  ncats,
  thresh_m,
  item_ind,
  dim_ind_i,
  refdim = 1,
  alpha = 1
)
```

```
lra2(dat, multi_pars, itemtype, item_ind, dim_ind_i, refdim = 1, alpha = 1)
```

**Arguments**

multi_pars	Vector of estimated multidimensional item parameters, to be rescaled.
uni_pars	Vector of estimated unidimensional item parameters, must be in the same order as multi_pars.
itemtype	Item type: "IPL", "PCM", or "PCM2", see <a href="#">align</a> .
item_ind	Vector with one element for each parameter indicating which item each parameter is associated with.
dim_ind_i	Vector with one element for each item indicating which dimension each item is associated with.
refdim	Which is the reference dimension (unchanged during alignment)

alpha	Vector of dimension steepnesses (often set equal to 1). Recycled if alpha is of length 1.
ss	Item sufficient statistics as proportions of examinees who reach each step, see <a href="#">align</a> . Should be of the same length as multi_pars.
nobs	Number of observed (non-missing) data points for each item.
ncats	Number of response categories for each item.
thresh_m	Vector of Thurstone thresholds. Should be the same length as multi_pars and ss.
dat	Data set with items as columns and examinees as rows. This data set must be the same data set used to estimate item parameters.

### Details

See [align](#).

### Value

List with the following elements

rhat	Vector of estimated scaling parameters $r$
shat	Vector of estimated shift parameters $s$
alphatilde	Vector of transformed dimension steepnesses
new_pars	Vector of transformed (i.e., aligned) item parameters
thresh	Vector of aligned Thurstone thresholds

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get_ss	<i>Get sufficient statistics</i>
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### Description

Find sufficient statistics from a data set

### Usage

```
get_ss(dat)
```

### Arguments

dat	Data set with items as columns and examinees as rows. Missing responses should be coded as NA.
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### Value

Vector of sufficient statistics



**Examples**

```

set.seed(2524)

diff_1 <- rnorm(10)
diff_2 <- rnorm(10)

N <- 500

th <- MASS::mvrnorm(N, mu = c(0, -1),
                    Sigma = matrix(c(1, .5 * 2, .5 * 2, 4), nrow = 2))

probs_1 <- 1 / (1 + exp(-outer(th[, 1], diff_1, "-")))
probs_2 <- 1 / (1 + exp(-outer(th[, 2], diff_2, "-")))

probs <- cbind(probs_1, probs_2)

dat <- apply(probs, 2, function(p) as.numeric(p > runif(N)))

get_ss(dat)

```

---

get\_thresh

*Get Thurstone thresholds*


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**Description**

Find Thurstone thresholds from a fitted model.

**Usage**

```
get_thresh(pars, itemtype, item_ind, alpha = 1)
```

**Arguments**

pars	Vector of estimated item parameters
itemtype	Item type: "IPL", "PCM", or "PCM2".
item_ind	Vector with one element for each parameter indicating which item each parameter is associated with.
alpha	Vector of item steepnesses, with one element for each item. Recycled if of length 1.

**Value**

Vector of Thurstone thresholds

**Examples**

```
if(require(TAM)){  
  set.seed(2524)  
  diff <- rnorm(10)  
  N <- 500  
  th <- rnorm(N)  
  probs <- 1 / (1 + exp(-outer(th, diff, "-")))  
  dat <- apply(probs, 2, function(p) as.numeric(p > runif(N)))  
  # fit the model  
  mod <- TAM::tam.mml(resp = dat, irtmodel = "1PL")  
  get_thresh(mod$xsi$xsi, itemtype = "1PL", item_ind = 1:10)  
}
```

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